Mathematical Logic and the Alvey Programme

The argument:

1. Research in mathematical logic is relevant to the intellectual problems of the Alvey Programme.

2. The teaching of mathematical logic at University level is relevant to the manpower problems of the Alvey Programme.

3. The present level of support for teaching and research in logic in the United Kingdom is well below that found in the United States.

4. If British universities in the face of their many difficulties are to move rapidly to meet the logic needs of the Alvey Programme, they will need some external stimulus.

5. As the increase of teaching and research in logic will further the Alvey programme, the funds at the disposal of the Alvey Directorate and the asociated industrialists may legitimately be applied to obtain such an increase.

6. An efficient and rapid mechanism for the strengthening of logic in the United Kingdom will be the establishment of Institutes of Logic on the lines proposed by the Logic Trust.

Introduction

1. The purpose of this document is to draw attention to the ways in which the aims of the Alvey Programme would be furthered by supporting teaching and research in mathematical logic. Reference will be made to the followng publications:

[A] "A Programme for Advanced Information Technology" (report of the DOI committee chaired by Mr J. Alvey, CB; Her Majesty's Stationery Office, October 1982)

[B] "Improving research links through Higher Education and Industry", (report of an ACARD–ABRC Working Group chaired by Sir Alan Muir Wood, FEng, FRS; Her Majesty's Stationery Office, June 1983)

[C] "The support given by Research Councils for in-house and university research" (report of an ABRC Working Party chaired by Mr J.R.S.Morris Feng, FIChemE; Her Majesty's Sationery Office, July 1983)

[D] "The Human Factor—The Supply Side Problem" (first report of the IT Skills Sortages Committee, chaired by Mr John Butcher M.P.; Department of Trade and Industry, July 1984)

[E] "Crisis facing UK Information Technology" (report of the IT EDC under Professor J.M. Ashworth, Chairman; National Economic Development Office, August 1984)

2. The Alvey Programme is a Government Initiative which seeks to foster the Information Technology Industry in the U.K. The report [A] of the Alvey Committee identifies four fundamental enabling technologies to which support should be given: software engineering (SE), man-machine interface (MMI), very large scale integration (VLSI), and intelligent knowledge-based systems (IKBS).

3. Mathematical logic is a subject floating between mathematics, philosophy, linguistics and computer science. The core of the subject is a concern with language: the mental characteristic of logicians is the ability to treat the sentences of a language as mathematical objects.

The techniques of logic are mathematical in character, and thus aim to be objective, impersonal and precise.

The most famous application of this psychological capacity for detachment from a language is perhaps Gödel's proof of his Incompleteness Theorem, published in 1931. A sense of the growth of logic in recent years may be obtained by leafing through "A Handbook of Mathematical Logic", edited by Jon Barwise (North Holland 1977).

4. Four ways in which logic may be relevant to IT can be identified:

- the study of logic may form a suitable academic preparation at undergraduate or graduate level for those wishing to pursue a career in IT.
- the results of logic may be ones that workers in IT might wish to apply.
- the underlying perceptions and methods of logic may provide a mode of thought, a way of approaching problems, a body of techniques that may profitably be adopted by those working on abstract problems in IT.
- there may be problems arising in IT which have abstract counterparts forming a direct challenge to logicians, who will thus work towards their solution.

These four ways are perhaps just points on a spectrum: in the first, logic is only a static source for IT; its dynamic involvement with IT increases through the next two, till in the last it takes the intellectual initiative.

5. Logic is related in varying degrees to all four enabling technologies.

6. Its relationship to SE is manifest in the design of programming languages, such as the new logic programming and functional programming languages; in the development of verification systems for imperative programming languages such as Ada; and generally in the trend towards a structured style of programming.

This trend arises from the fact that large programs cannot be verified by hand; their correctness, though, must be established somehow, as computers, being essentially stupid, are much more upset by mistakes in what they read than are people; consequently if practical programs are to be clear and clean, they must be linked to mathematical counterparts which can be formally tested by mathematical means.

A 4.2.18(8) calls for support of research in areas such as very high level language and language theory.

7. The relationship of logic to MI and to IKBS is brought out in sections 4.3 and 4.4 of the Alvey report, where many of the ideas are related to those in Professor Tennant's letter to the Director of the Alvey Programme, in which he proposes the establishment of an Institute of Logic and Cognitive Studies at Stirling:

> *"in addition to work in artificial intelligence, there is much to be done in cognitive science, in logic and in semantics ..."* [A 4.4.4]

> *"...The way forward appears to lie in the use of natural languages ..."* [A 4.4.6]

8. The original role of logic in the design of hardware was confined to the simple Boolean algebra of logic gates; but the relationship of logic to VSLI is growing in importance because the testing of the design of the modern densely-packed chip must proceed by software simulation.

9. As examples of the relevance of past work in logic to contemporary work in computer science, one may cite Curry's work from 1930 onwards on combinatorial logic, where the completeness theorem may today be interpreted as a theorem about the sufficiency of compilers; Church's work, starting in 1932, on the lambda calculus, which is now the basis of denotational semantics and of functional programming languages like LISP; the work on unification theory, which is foreshadowed in Herbrand's thesis of 1929; and applications of Kleene's work of the 1940s on realizability to the design of experimental programming languages such as PRL which aim to have so close a relationship between a program and its verification that, to quote the proponent's slogan, "The proof is the program".

One area where abstract computer science meets contemporary logic is that of complexity theory, centring round attempts to solve the P=NP problem.

Among researchers in AI there is a belief that self-reference must be an important aspect of any worthwhile AI system: witness the self-referential aspects of LISP, which was designed at least in part for AI. Such self-reference has evident connections with logic.

10. These examples illustrate the first point, that **research in mathematical logic is relevant to the intellectual problems of the Alvey programme.**

The enlargement of the IT Community

11. The Alvey report is visionary and nationalistic, and its underlying message is sociological. The report seeks to express the unwelcome fact that Britain is, in terms of IT, becoming a backward country. It draws attention to the fragmented character of the British research effort, to the manpower shortage, to the importance of research, and to the unattractive conditions Britain offers to researchers compared with elsewhere. It presents a very important conclusion: that if Britain is not simply to be an outpost of the IT communities of other countries, but is to establish her own vigorous IT character, what is needed is a sociological change, and it calls for such change as a specific aim of the Alvey programme, to be actively pursued and not just passively desired:

"The objective is to create a new community." [A 4.6.1]

12. Part of the problem of strengthening and re-structuring the IT community of Britain is the fact that the community is actually too small. There is a manpower problem:

"IT is knowledge intensive and dependent on skilled manpower." [A 7.1]

"There must be a substantial increase in the number of people trained in IKBS research, development and production." [A 4.4.9]

"In 2–3 years the IKBS community should increase by about 50%, chiefly by attracting interested workers from bordering fields." [A 4.4.16a]

The existence of a manpower problem is affirmed by the IT EDC document [E].

13. The Alvey report recognises that to achieve this increase in the IT community organically one must seek to attract young people at Universities and other HEIs, and consequently

"This means urgent action in the higher education sector." [A 7.3]

It is then natural to ask what education will constitute a suitable preparation for a career in IT.

14. There is a clear answer to this question on page 298 of *The Science of Programming* by David Gries, Professor of Computer Science at Cornell University (Springer-Verlag New York, Heidelberg, Berlin)

> *"The research* [on axiomatic definitions in the 1970s] *was fraught with lack of understanding and frustration. One reason for this was that computer scientists in the field, as a whole, did not know enough formal logic. Some papers were written simply because the authors didn't understand earlier work; others contained errors that wouldn't have happened had the authors been educated in logic. ... We spent a good deal of time thrashing, just treading water, instead of swimming, because of our ignorance. With hindsight, I can say that the best thing for me to have done 10 years ago would have been to take a course in logic. I persuaded many students to do so, but I never did so myself."*

15. In view of the strong links between the abstract superstructure of IT and the concerns of mathematical logic, and the consequential ability of work in logic to be a source of ideas and techniques applicable to the difficult abstract problems arising in IT, it is suggested that an important element of an educational preparation for a career in IT will be a training in logic.

16. There is a further aspect of this which pertains to the quality and not the quantity of the IT intake. Of course there are careers in IT at a variety of intellectual levels; but since among the problems in IT are many very difficult ones,

"The research required is not only difficult, it is essentially long-term" [A 4.4.4]

it is to be hoped that among the IT intake will be young graduates with first class minds. Now clever undergraduates with a particular understanding of mathematics are attracted to logic as an abstract discipline; it is therefore desirable for links between logic and IT to be visible at University level, so that these first class people will see IT as offering careers giving full scope to the exercise of their talents on intellectually challenging problems.

17. This might be the place to repeat a general point about the virtues of a training in abstract thought even for those whose professional lives will be lived at a more practical level. In brief, the drawback to too great an emphasis on "modern techniques" at the expense of general, but abstract, principles, is that within

a few years, the "modern techniques" will be outmoded whereas the general principles will remain valid. Consequently, though the work in IT will be largely "downstream", it is desirable that much of the education for it takes place "upstream".

At Cornell University, for example, computer scientists do two full years of logic, including set theory and model theory, as part of their course; and methods of building technology into the teaching of logic are being evolved.

18. These considerations suggest that **the teaching of mathematical logic at University level is relevant to the manpower problems of the Alvey programme.**

The position of logic in the United Kingdom

19. Thus it is in the interests of the Alvey Programme that there should be a strong teaching programme in logic, and hence an appraisal must be made of the extent to which universities are able to provide the sort of support for logic that would meet the needs of the Alvey Programme.

20. The concern felt by the Alvey Committee stems from a comparison of Britain's position with that of other countries. When one now looks at the opportunities for receiving a training in logic and for an academic career in logic in the U.K., and compares them with those in the U.S., one finds that the situation is grave. Indeed, there is the same sense among British logicians of the logic community in Britain being "fragmented" as has been remarked by the Alvey Committee about the British IT community.

21. It was perceptions of this kind that led to the formation of the Logic Trust, a registered charity which, when its funds permit, has as its aim the advancement of teaching and research in logic through the establishment of institutes of logic.

22. The Logic Trust has prepared a table comparing the number of lectures given at undergraduate and graduate levels at Cambridge with the number given at leading American Universities: at Cambridge a student mathematician might hope to hear 50 lectures in the course of 4 years; whereas at Princeton, Yale, Harvard ... he might hear, on average, 250; and at places like Berkeley where there is a strong specialisation in logic, he may hear 400.

Even after adjusting these figures to allow for the difference between the university systems of the two countries, there remains a gross discrepancy in the sheer opportunity of learning.

23. There is no shortage of interest in logic among undergraduates at British universities: but the prospect of employment in a British university for those who go on to obtain a Ph. D. in logic is bleak, since, for example, the proportion of posts in the Mathematics Department of Cambridge held by logicians is a quarter of the proportion in, say, CalTech. Consequently, holders of Ph.D.s in logic must look elsewhere: of nine recent recipients of British Ph.D.s in logic, seven are now teaching at American universities, one in Canada, and one in New Zealand.

24. These statistics illustrate the fact that **the present level of support for teaching and research in logic in the United Kingdom is well below that found in the United States.**

The difficulties facing British Universities

25. So far it has been suggested that a significant contribution to the Alvey programme would result from teaching and research in mathematical logic, but that logic in the U.K. is below strength. It is therefore appropriate to consider to what extent universities will themselves be able to move to meet the logic needs of the Alvey programme.

26. There has been much discussion ([A], [B], [C], [D]) recently of the relationship of universities and industry and of the problem of funding industry-oriented research at universities.

27. Common to these reports is a general expectation that initiative in meeting the manpower needs of industry will or should come from universities.

Thus the Butcher report [D] says that there is a great shortage in industry of mathematicians and computer scientists; the UGC should identify their teaching needs; and industry should then meet the expense.

The same supposition holds when research rather than teaching needs are being discussed; for example in .4 of the Summary of the Muir Wood report [B], it is suggested that the initiative should lie mainly with

the HEIs; and when HEIs show initiative, industry should be ready to respond.

Similarly the Morris report [C] maintains a belief in universities taking the initiative even in industry-oriented research:

> *"We recommend that each university should have a Research Committee."* [C 59]

> *"A substantial proportion of the funds remaining ... should be disbursed by a research committee of the university."* [C 148]

28. There is a perception in these reports of the importance of good leadership:

> *"The significant UK investment in research depends for its success on knowledgeable people with lively enquiring and frequently unorthodox minds who are stimulated by good leadership and supported by appropriate facilities."* [C 135]

> *"The research councils, etc. provide specialist and concentrated research facilities which ... would be difficult to establish in a university. The scientific leadership invariably has the task of counteracting the natural tendency of such institutions to become inward looking and insufficiently aware of the available research resources of the universities."* [C 137]

29. But in spite of the apparent expectation, or hope, that universities and research councils will take the initiative, there is also a perception that they are impeded by difficulties, and consequently acting in a timid, confused, and therefore disappointing way:

> *"This has exacerbated a situation where academic researchers are already overburdened by teaching and administrative duties and frsutrated by the difficulty of getting research funding."* [A 7.5]

> *"The rôle of the tertiary education sector in relation to computing is currently confused."* [A 7.10]

> *"We found evidence of a belief that Councils were too inclined to support projects in "safe" areas of well-known research activity ..."* [C 40]

> *"It is evident that many constraints are at the moment reducing the effectiveness with which universities engage in research. These have largely arisen from the financial economies imposed by the Government ..."* [C 55]

> *"...the reduction in UGC funds combined with the difficulty of reducing academic staff numbers put on universities an irresistible pressure to reduce the funds they provided towards the recurrent costs of research."* [C 58]

> *"The research potential in universities is to some extent under-utilised because of staff time and funding constraints."* [C 136]

30. It is clear that Universities are under financial pressure; earlier this century direct funding from government was the norm; the present government is seeking to move from that to a position similar to American practice, where much funding of university research comes from industry.

The following comparison may illustrate the implications of this: CalTech has a team of forty people (which they consider to be on the small side compared with those of other universities) whose sole concern is building links with industry, with the intention that money should come from industry for research at CalTech, and that conversely the research done at CalTech should be of relevance to industry. The only comparable body at Cambridge seems to be the recently established Wolfson Industrial Unit, under Dr S. L. Bragg, which aims to foster links between the University and research.

31. This financial pressure means that within any university the internal struggle for funds will make it politically unlikely that subjects that are strongly placed in the university will willingly give up part of their budget to subjects such as logic which have hitherto only been weakly supported.

32. In brief, a university is likely to allocate its resources according to its present internal values and therefore to perpetuate them. Just as Britain is only being led to an IT initiative by a vision based on the external evidence of comparison with other nations, and not by any internal reason, so a university is unlikely to create a new teaching programme in logic for purely internal reasons. Some external event will be necessary before such changes happen.

33. In fairness to British Universities, it must be said that they have been seriously undernourished for some time. Not only are academic salaries meagre compared with those in the U.S., where full professors at Ivy League universities have salaries that place them in the upper 0.5% of the salaried classes, but the teaching loads are much heavier: at Cambridge, a lecturer in mathematics is typically expected to give six hours of lectures and six hours of undergraduate supervision a week, making twelve hours. At CalTech, the teaching load is three hours a week; moreover, a professor may arrange his year so as to give six hours teaching one term and none the next, thus permitting a greater concentration on research.

34. Concern is expressed in the Butcher report [D] that British experts who have emigrated should be attracted back to this country. But reversing the brain drain is not merely a question of offering higher salaries. A consideration, even more important than that of pay, that has commonly led to a decision by an academic to emigrate is the perception that the present high teaching and administrative burden imposed by his British university is not merely reducing his freedom for research but threatening his actual capacity for it.

35. For all these reasons it is to be feared that **if British Universities in the face of their many difficulties are to move rapidly to meet the logic needs of the Alvey Programme, they will need some external stimulus.**

Alvey funds and the support of logic

36. Thus it would serve the aims of the Alvey programme for Alvey money to be devoted directly to the support of teaching and research in logic in the United Kingdom. The general question of the legitimacy of this and similar expenditure therefore arises.

37. The original Alvey report takes an affirmative view:

> *"Some £57m should go to support research and training in academic institutions. This should be 100% Government funded."* [A 2.4]

> *"More teaching posts must be created; more research appointments, including fellowships and assistantships of all kinds must be funded."* [A 4.4.11]

> *"The programme also includes a substantial amount of research to be conducted in academic establishments. This should be funded by SERC."* [A 5.4]

38. It has been suggested, though, that the industrialists associated with the Alvey programme would be reluctant to give direct support to a subject apparently so far upstream from the practical concerns of the Alvey programme as mathematical logic.

39. Against that, it might be argued that the many examples in the history of science of upstream work coming downstream in time suggest that such reluctance is mistaken, and out of line with the adventurous and long-sighted spirit of the Alvey initiative.

40. To invite further discussion and clarification of this point, it is therefore now suggested that **as the increase of teaching and research in logic will further the Alvey programme, the funds at the disposal of the Alvey Directorate and the associated industrialists may legitimately be applied to obtain such an increase.**

The establishment of Institutes of Logic

41. The goal of the Alvey programme, however, is not the solution of the problems of the large and slow-moving bodies that are our universities, but the establishment of the British IT industry against foreign competition. The difficulties facing our universities, therefore, while undeniably of national importance, are in the main not the concern of the Alvey Directorate, who, operating within a limited timescale, must seek ways of sidestepping them.

42. What is wanted, therefore, is a rapid and efficient way of initiating conjoined programmes of teaching and research in logic, with the interlocking purposes of attracting undergraduates of high calibre to a career in IT by enabling them to see that there are intellectually challenging problems in the field; giving these potential recruits to the IT community a thorough training in logic; and contributing to the solution of the theoretical problems arising in IT.

43. It is therefore suggested that rather than ask universities to establish suitable programmes of teaching and research in logic, which for the reasons stated they would be very slow to do, Logic Institutes should be created and then offered, on some co-operative basis, to universities, who would be very quick to accept.

44. Some of the ideas about Logic Institutes that motivated the formation of the Logic Trust are echoed by the Alvey report:

> *"We propose that a national IT Fellowship be established, ... The IT Fellowship is an integral part of our total programme."* [A 7.7]

> *"There is a real opportunity here for a "Distributed College" to emerge, linking information technologists in industry and the academic sector."* [A 7.13]

45. Specific proposals for the design and establishment of such Institutes at Cambridge and at Stirling are in existence. The simplicity of the administrative structure envisaged for these Institutes would assist their growth as centres of research. It would, though, be premature, if tempting, for the present document to go into the details of the courses that a Logic Institute might develop in co-operation with the local mathematicians, philosophers, and computer scientists.

46. Thus it is suggested that **an efficient and rapid mechanism for the strengthening of logic in the United Kingdom will be the establishment of Institutes of Logic on the lines proposed by the Logic Trust.**

[1.ii.1985]