COLLOQUIUM MATHEMATICUM

VOL. XXXVIII 1977 FASC. 1

A SOLUTION OF A PROBLEM OF B. ROTMAN

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The answer to P 937 (1), correcting an earlier paper, is negative.

THEOREM. *Let M be an infinite set of power a, and $f_i\colon M \to M$ an injection for each $i < a$. The following statements are equivalent*:

(i) *there is a subset $X \subseteq M$ of power a such that any two members of the sequence*

$$X, f_0(X), f_1(X), \ldots, f_i(X), \ldots \qquad (i < a)$$

are almost disjoint;

(ii) *there is a subset $Z \subseteq M$ of power a such that, for all i and $j < a$, $F_{ij} \cap Z$ and $F_i \cap Z$ have power less than a, where*

$$F_i = \{x \in M : f_i(x) = x\} \quad \text{and} \quad F_{ij} = \{x \in M : f_i(x) = f_j(x)\}.$$

Proof. For (ii) $\Rightarrow$ (i), repeat the proof of Rotman's theorem (op. cit.).

For (i) $\Rightarrow$ (ii), take $Z = X$.

So, for a counterexample to P 937, suppose that f_1, f_2, and f_3 are such that, for all x in M, $f_3(x) = f_1(x)$ or $f_3(x) = f_2(x)$. Then, for any Z of power a, either $Z \cap F_{13}$ or $Z \cap F_{23}$ is of power a. So (ii), and hence (i) will be false.

A necessary and, as there are a functions, sufficient condition for (ii) when a is regular is that the a-ideal generated by the F_i, the F_{ij} and the small subsets of M should be proper.

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(1) B. Rotman, *Correction to the paper "A theorem on almost disjoint sets"*, Colloquium Mathematicum 32 (1975), p. 307-308.

Reçu par la Rédaction le 4. 5. 1976