Correction to [1]

There is an error in the formulae of §3 of [1], in the case when x is a point where $j = 0$ or 1728 and $\kappa(x) \neq \mathbb{F}_p$. The formulae which need correction are (3.2) and the unnumbered formula for $j = 1728$, $e(x) = 2$ and $p \equiv 1 \pmod 4$. (The case $\kappa(x) = \mathbb{F}_p$, which is the only case used in the computational example which was the object of the paper, is unaffected.) It is simplest just to rederive these formulae for all p, and not to bother to write the trace in terms of cubic or quartic residue symbols over finite fields.

We can rewrite the "generic j" curve after a change of variables as

$$3y^2 = x^3 + 3x^2 + \frac{3x+1}{1-12^{-3}j}.$$

Case $\pi(x) = 1728$, $e(x) = 2$.

Write $1 - 12^{-3}j = u\varpi^2$. In the genus 0 case with j-equations as in (2.3), u is the leading coefficient of the expansion of $(Q - 12^{-3}P)/Q$ about $t = t_0$.

After a change of variables the curve becomes

$$3\varpi y^2 = x^3 + 3u^{-1}x + \varpi(3x^2 + u^{-1})$$

so the local trace is

$$\sum_{-\frac{k}{2} \leq i \leq \frac{k}{2}} q^{k/2} (\alpha/\bar{\alpha})^i \qquad (*)$$

as in §3.2, where $1 + q - \alpha - \bar{\alpha}$ is the number of $\mathbb{F}_q$-points on the curve

$$y^2 = x^3 + 3u^{-1}x.$$

Case $\pi(x) = 0$, $e(x) = 3$.

Write $1 - 12^{-3}j = 1 - u\varpi^3$. In terms of j-equations, u is the leading coefficient of $12^{-3}j = 12^{-3}P/Q$ about $t = t_0$.

This time the equation becomes

$$3\varpi y^2 = x^3 - 3\varpi ux + 2u$$

and so the local trace is given by formula $(*)$ applied to the curve

$$y^2 = x^3 + 2u.$$

References

[1] A. J. Scholl: *The ℓ-adic representations associated to a certain noncongruence subgroup*. J. für die reine und ang. Math. **392** (1988), 1–15