Part III: Riemannian geometry. **A.G. Kovalev. Lent 2017**

Final schedule

- Algebraic symmetries of the Riemann curvature. Ricci and scalar curvature. Sectional curvature. Examples (constant sectional curvature, invariant metrics on Lie groups). Covariant derivative along a curve.
- Geodesics, the exponential map. Gauss lemma. Jacobi fields. Length-minimizing geodesics.
- Completeness of a Riemannian manifold. Hopf–Rinow theorem. Variations of length and of energy.
- Synge's theorem. Conjugate points. Bonnet–Myers diameter theorem. Riemannian coverings. Hadamard–Cartan theorem.
- Volume form and the Hodge star operator. Laplace–Beltrami operator. The Hodge decomposition theorem (assuming the compactness, regularity and Hahn–Banach theorems). Harmonic forms and the de Rham cohomology.
- Divergence of a vector field on Riemannian manifold. Bochner–Weitzenböck formula for 1-forms, application to Ricci curvature and the de Rham cohomology.
- Holonomy groups, the fundamental principle of Riemannian holonomy. Holonomy algebra and curvature. Holonomy and the de Rham cohomology.
- Rays and lines in a Riemannian manifold. The Cheeger–Gromoll splitting theorem (statement only). The fundamental group and non-negative Ricci curvature.

Example sheets form an official part of this course and the exam questions may contain short problem elements similar to what appeared in the examples (*not* those marked with *).