

1. Write down subsets of the reals that have order-types $\omega + \omega$, ω^2 and ω^3 .
2. Let α and β be non-zero ordinals. Must we have $\alpha + \beta > \alpha$? Must we have $\alpha + \beta > \beta$?
3. Is there a non-zero ordinal α with $\alpha\omega = \alpha$? What about $\omega\alpha = \alpha$?
4. Show that the inductive and the synthetic definitions of ordinal multiplication coincide.
5. Let α, β, γ be ordinals. Prove that $(\alpha\beta)\gamma = \alpha(\beta\gamma)$.
6. Let α, β, γ be ordinals. Must we have $(\alpha + \beta)\gamma = \alpha\gamma + \beta\gamma$? Must we have $\alpha(\beta + \gamma) = \alpha\beta + \alpha\gamma$?
7. Let α and β be ordinals with $\alpha \geq \beta$. Show that there is a unique ordinal γ such that $\beta + \gamma = \alpha$. Must there exist an ordinal γ with $\gamma + \beta = \alpha$?
8. An ordinal written as $\omega^{\alpha_1}n_1 + \dots + \omega^{\alpha_k}n_k$, where $\alpha_1 > \dots > \alpha_k$ are ordinals (and k and n_1, \dots, n_k are non-zero natural numbers), is said to be in *Cantor Normal Form*. Show that every non-zero ordinal has a unique Cantor Normal Form. What is the Cantor Normal Form for the ordinal ϵ_0 ?
9. Is ω_1 a successor or a limit?
10. Let α be a countable (non-zero) limit ordinal. Prove that there exists an increasing sequence $\alpha_1 < \alpha_2 < \alpha_3 < \dots$ with supremum equal to α . Is this result true for $\alpha = \omega_1$?
11. Show that, for every countable ordinal α , there is a subset of \mathbb{Q} of order-type α . Why is there no subset of \mathbb{R} of order-type ω_1 ?
12. Let X be a totally ordered set such that every subset of X is isomorphic to some initial segment of X . Prove that the ordering must be a well-ordering.
13. Is it possible to select for each countable (non-zero) limit ordinal α an ordinal $x_\alpha < \alpha$ in such a way that the x_α are distinct?
- +14. Let X be a totally ordered set such that the only order-preserving injection from X to itself is the identity. Must X be finite?