

1. For which m and n is the complete bipartite graph $K_{m,n}$ Hamiltonian? Is the Petersen graph $(\{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}, \{01, 12, 23, 34, 40, 57, 79, 96, 68, 85, 05, 16, 27, 38, 49\})$ Hamiltonian?
2. What is $\text{ex}(P)$, where P is the Petersen graph?
3. For each $r \geq 3$, construct a graph of chromatic number r that contains no K_r .
4. Show that a graph with no odd cycles must be bipartite.
5. Let n, k be positive integers with $k < n/2$. Show that a connected graph G with $|G| = n$ and $\delta(G) \geq k$ must contain a path of length $2k$.
6. Show that $\text{ex}(n; K_{2,t}) \leq \frac{n}{4} \left(1 + \sqrt{1 + 4(n-1)(t-1)}\right)$.
7. What is $\text{ex}(n; K_{1,t})$?
8. Let G be a graph of order $n \geq 5$ with $e(G) \geq \lfloor \frac{n^2}{4} \rfloor + 2$. Show that G must contain two triangles with precisely one vertex in common.
9. Let x_1, x_2, \dots, x_{3n} be points in the plane such that no two of them are more than distance 1 apart. Prove that at most $3n^2$ of the distances $\|x_i - x_j\|$ ($i < j$) are greater than $1/\sqrt{2}$.
10. Without assuming the Erdős-Stone theorem, show that $\text{ud}(G)$ is well-defined for all infinite graphs G . That is, show that $\lim_{n \rightarrow \infty} \sup\{D(H) : H \subset G, |H| = n\}$ exists.
11. For each positive integer n , let g_n be the largest integer k such that it is possible to colour k edges of the complete graph K_n blue or yellow without creating a monochromatic (blue or yellow) triangle. Show that $g_n/\binom{n}{2}$ converges, and find $\lim_{n \rightarrow \infty} g_n/\binom{n}{2}$.
12. How many edges can a graph G of order n containing precisely one triangle have?
13. Let $x_1, x_2, \dots \in \mathbb{R}^d$ be distinct and, for each n , let $f(n)$ be the number of pairs of points from x_1, \dots, x_n that are at distance 1: $f(n) = |\{(i, j) : 1 \leq i < j \leq n, \|x_i - x_j\| = 1\}|$. Show that if $d = 3$ then $f(n)/\binom{n}{2} \rightarrow 0$ as $n \rightarrow \infty$. Show more generally that if $d \geq 3$ then $f(n) \leq (1 - \frac{1}{D} + o(1))\binom{n}{2}$, where $D = \lfloor d/2 \rfloor$.
- ⁺14. Construct a triangle-free graph of chromatic number 1526.
- ⁺15. Show that an r -regular graph of order $2r + 1$ must be Hamiltonian.