

1. For which  $m$  and  $n$  is the complete bipartite graph  $K_{m,n}$  Hamiltonian? Is the Petersen graph  $(\{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}, \{01, 12, 23, 34, 40, 57, 79, 96, 68, 85, 05, 16, 27, 38, 49\})$  Hamiltonian?
2. What is  $\text{ex}(P)$ , where  $P$  is the Petersen graph?
3. For each  $r \geq 3$ , construct a graph of chromatic number  $r$  that contains no  $K_r$ .
4. Show that a graph with no odd cycles must be bipartite.
5. Let  $n, k$  be positive integers with  $k < n/2$ . Show that a connected graph  $G$  with  $|G| = n$  and  $\delta(G) \geq k$  must contain a path of length  $2k$ .
6. Show that  $\text{ex}(n; K_{2,t}) \leq \frac{n}{4} \left(1 + \sqrt{1 + 4(n-1)(t-1)}\right)$ .
7. What is  $\text{ex}(n; K_{1,t})$ ?
8. Let  $G$  be a graph of order  $n \geq 5$  with  $e(G) \geq \lfloor \frac{n^2}{4} \rfloor + 2$ . Show that  $G$  must contain two triangles with precisely one vertex in common.
9. Let  $x_1, x_2, \dots, x_{3n}$  be points in the plane such that no two of them are more than distance 1 apart. Prove that at most  $3n^2$  of the distances  $\|x_i - x_j\|$  ( $i < j$ ) are greater than  $1/\sqrt{2}$ .
10. Without assuming the Erdős-Stone theorem, show that  $\text{ud}(G)$  is well-defined for all infinite graphs  $G$ . That is, show that  $\lim_{n \rightarrow \infty} \sup\{D(H) : H \subset G, |H| = n\}$  exists.
11. For each positive integer  $n$ , let  $g_n$  be the largest integer  $k$  such that it is possible to colour  $k$  edges of the complete graph  $K_n$  blue or yellow without creating a monochromatic (blue or yellow) triangle. Show that  $g_n/\binom{n}{2}$  converges, and find  $\lim_{n \rightarrow \infty} g_n/\binom{n}{2}$ .
12. How many edges can a graph  $G$  of order  $n$  containing precisely one triangle have?
13. Let  $x_1, x_2, \dots \in \mathbb{R}^d$  be distinct and, for each  $n$ , let  $f(n)$  be the number of pairs of points from  $x_1, \dots, x_n$  that are at distance 1:  $f(n) = |\{(i, j) : 1 \leq i < j \leq n, \|x_i - x_j\| = 1\}|$ . Show that if  $d = 3$  then  $f(n)/\binom{n}{2} \rightarrow 0$  as  $n \rightarrow \infty$ . Show more generally that if  $d \geq 3$  then  $f(n) \leq (1 - \frac{1}{D} + o(1))\binom{n}{2}$ , where  $D = \lfloor d/2 \rfloor$ .
- <sup>+</sup>14. Construct a triangle-free graph of chromatic number 1526.
- <sup>+</sup>15. Show that an  $r$ -regular graph of order  $2r + 1$  must be Hamiltonian.