1. Show that $R(3,4) \leq 9$. By considering the graph on $\mathbb{Z}_8$ (the integers modulo 8) in which x is joined to y if $x - y = \pm 1$ or ± 2, show that $R(3,4) = 9$.

2. By considering the graph on $\mathbb{Z}_{17}$ in which x is joined to y if $x - y$ is a square modulo 17, show that $R(4,4) = 18$.

3. Show that $R_3(3,3,3) \leq 17$.

4. Let A be a set of $R^{(4)}(n,5)$ points in the plane, with no three points of A collinear. Prove that A contains n points forming a convex n-gon.

5. Let A be an infinite set of points in the plane, with no three points of A collinear. Prove that A contains an infinite set B such that no point of B is a convex combination of other points of B.

6. Show that every graph G has a partition of its vertex-set as $X \cup Y$ such that the number of edges from X to Y is at least $\frac{1}{2}e(G)$. Give three proofs: by induction, by choosing an optimal partition, and by choosing a random partition.

7. In a *tournament* on n players, each pair play a game, with one or other player winning (there are no draws). Prove that, for any k, there is a tournament in which, for any k players, there is a player who beats all of them. [Hint: consider a random tournament.] Exhibit such a tournament for $k = 2$.

8. Let X denote the number of copies of K_4 in a random graph G chosen from $G(n,p)$. Find the mean and the variance of X. Deduce that $p = n^{-2/3}$ is a threshold for the existence of a K_4, in the sense that if $pn^{2/3} \to 0$ then almost surely G does not contain a K_4, while if $pn^{2/3} \to \infty$ then almost surely G does contain a K_4.

9. Find the eigenvalues of K_n. Find the eigenvalues of $K_{n,m}$.

10. Prove that the matrix J (all of whose entries are 1) is a polynomial in the adjacency matrix of a graph G if and only if G is regular and connected.

11. Let G be a graph in which every edge is in a unique triangle and every non-edge is a diagonal of a unique 4-cycle. Show that G is k-regular, for some k, and that the number of vertices of G is $1 + k^2/2$. Show also that k must belong to the set $\{2, 4, 14, 22, 112, 994\}$.

12. Let the infinite subsets of $\mathbb{N}$ be 2-coloured. Must there exist an infinite set $M \subset \mathbb{N}$ all of whose infinite subsets have the same colour?

$^+$13. Let A be an uncountable set, and let $A^{(2)}$ be 2-coloured. Must there exist an uncountable monochromatic set in A?