- 1. For which n and m is the complete bipartite graph $K_{n,m}$ Hamiltonian? Is the Petersen graph Hamiltonian?
- 2. Let G be a graph of order n with $e(G) > \binom{n}{2} (n-2)$. Prove that G is Hamiltonian.
- 3. Let G be a bipartite graph with vertex classes X, Y. Show that if G has a matching from X to Y then there exists $x \in X$ such that every edge incident with x extends to a matching from X to Y.
- 4. Let G be a connected bipartite graph with vertex classes X,Y. Show that every edge of G extends to a matching from X to Y if and only if $|\Gamma(A)| > |A|$ for every $A \subset X$, $A \neq \emptyset, X$.
- 5. Let A be a matrix with each entry 0 or 1. Prove that the minimum number of rows and columns containing all the 1s of A equals the maximum number of 1s that can be found with no two in the same row or column.
- 6. For $r \leq n$, an $r \times n$ Latin rectangle is an $r \times n$ matrix, with each entry from $\{1, \ldots, n\}$, such that no two entries in the same row or column are the same. Prove that every $r \times n$ Latin rectangle may be extended to an $n \times n$ Latin square.
- 7. Show that we always have $\kappa(G) \leq \lambda(G)$. For any positive integers $k \leq l$, construct a graph G with $\kappa(G) = k$ and $\lambda(G) = l$.
- 8. For a set $B \subset V(G)$ and a vertex a not in B, an a-B fan is a family of |B| paths from a to B, disjoint except at a. Show that a graph G (with |G| > k) is k-connected if and only if there is an a-B fan for every $B \subset V(G)$ with |B| = k and every vertex a not in B.
- 9. Let G be a k-connected graph $(k \geq 2)$, and let x_1, \ldots, x_k be vertices of G. Show that there is a cycle in G containing all the x_i .
- 10. For each $r \geq 3$, construct a graph G such that G does not contain K_r but G is not (r-1)-partite.
- 11. A deleted K_r consists of a K_r from which an edge has been removed. Show that if G is a graph of order n $(n \ge r + 1)$ with $e(G) > e(T_{r-1}(n))$ then G contains a deleted K_{r+1} .
- 12. Let x_1, \ldots, x_n be points in the plane such that no two of them are more than distance 1 apart. Prove that, of the $\binom{n}{2}$ possible pairs of points, at most $n^2/3$ are at distance greater than $1/\sqrt{2}$.
- +13. Let G be a (possibly infinite) bipartite graph, with vertex classes X, Y, such that $|\Gamma(A)| \geq |A|$ for every $A \subset X$. Give an example to show that G need not contain a matching from X to Y. Show however that if G is countable and $d(x) < \infty$ for every $x \in X$ then G does contain a matching from X to Y. Does this remain true if G is uncountable?
- $^{+}14$. Let G be an r-regular graph on 2r+1 vertices. Prove that G is Hamiltonian.