

# 2012 British Topology Meeting

## Titles and Abstracts

### Thursday, Sept. 6

**Speaker:** Stefan Bauer (Bielefeld)

**Title:** Seiberg-Witten maps and stable homotopy

Consider the space of all compact non-linear perturbations of a fixed linear Fredholm operator  $L$  satisfying a boundedness condition: Preimages of bounded sets are supposed to be bounded. Suitably topologized, this space will have the weak homotopy type of the infinite loop space  $\Omega^\infty \Sigma^\infty(S^{-ind(L)})$ . The Seiberg-Witten map for a closed four-manifold satisfies the necessary boundedness conditions and thus defines a map of spectra: From the Thom spectrum of the virtual index bundle of the Dirac operator over the Picard group of the manifold to the sphere spectrum. This way one obtains a lift of what is known as the Bauer-Furuta invariant from the homotopy category of spectra to the category of spectra. An extension of this construction to compact four-manifolds with boundary a finite union of rational homotopy three-spheres will be discussed, as well as consequences.

**Speaker:** Oscar Randal-Williams (Copenhagen)

**Title:** Stable moduli spaces of high dimensional manifolds

**Abstract:** I will discuss recent joint work with S. Galatius, in which we generalise the Madsen–Weiss theorem from the case of surfaces to the case of manifolds of higher even dimension (except 4). In the simplest case, we study the topological group  $\mathcal{D}_g$  of diffeomorphisms of the manifold  $\#^g S^n \times S^n$  which fix a disc. We have two main results: firstly, a homology stability theorem—analogueous to Harer’s stability theorem for the homology of mapping class groups—which says that the homology groups  $H_i(B\mathcal{D}_g)$  are independent of  $g$  for  $2i \leq g - 4$ . Secondly, an identification of the stable homology  $H_*(B\mathcal{D}_\infty)$  with the homology of a certain explicitly described infinite loop space—analogueous to the Madsen–Weiss theorem. Together, these give an explicit calculation of the ring  $H^*(B\mathcal{D}_g; \mathbb{Q})$  in the stable range, as a polynomial algebra on certain explicitly described generators.

**Speaker:** Simon Donaldson (Imperial)

**Title:** Progress and problems in geometry on  $G_2$  manifolds

**Abstract:** For more than a decade, it has been realised that it might be possible to develop “enumerative geometries” on 7-manifolds with holonomy  $G_2$ . One side of this putative theory considers submanifolds and the other considers gauge fields. There has been much progress recently in the general existence theory and understanding of examples, through work of Sa Earp, Nordstrom, Walpuski and others. This progress brings some central difficulties in defining enumerative invariants into clearer focus. These difficulties are largely of a topological nature. In the talk I will review the fundamental definitions, and attempt to outline the above-mentioned progress and difficulties.

### Friday, Sept. 7

**Speaker:** Marc Lackenby (Oxford)

**Title:** A polynomial upper bound on Reidemeister moves

**Abstract:** Consider a diagram of the unknot with  $c$  crossings. There is a sequence of Reidemeister moves taking this to the trivial diagram. But how many moves are required? In my talk, I will give an overview of my recent proof that there is an upper bound on the number of moves, which is a polynomial function of  $c$ .

**Speaker:** Richard Webb (Warwick)

**Title:** Tightness and acylindrical actions

**Abstract:** This talk will focus on the curve complex - a Gromov hyperbolic space that is used to study the mapping class group, Teichmueller space and infinite volume hyperbolic manifolds. We will give a new proof of the theorem that the action of the mapping class group on the curve complex is acylindrical (originally due to Bowditch). The acylindrical theorem, along with work of Dahmani-Guirardel-Osin, shows that for a fixed surface  $S$  there exists  $N$  such that the normal closure of  $f^N$ , where  $f$  is a pseudo-Anosov mapping class on  $S$ , is a free and purely pseudo-Anosov subgroup. Our methods for the proof of acylindricity are elementary and give explicit bounds. Along the way we also give new proofs that there are finitely many tight geodesics between any pair of vertices and we give a new distance algorithm for the curve complex.

**Speaker:** Nathalie Wahl (Copenhagen)

**Title:** Higher string topology

**Abstract:** We give a construction of non-trivial higher degree operations on the homology of the free loop space of a manifold associated to surfaces of any genus. These operations are parametrized by Sullivan diagrams, which model a partial compactification of the moduli space of Riemann surfaces.

**Speaker:** Paul Biran (ETH)

**Title:** Lagrangian topology and cobordisms

**Abstract:** Lagrangian submanifolds appear naturally in symplectic geometry from different angles: real algebraic geometry, dynamics and integrable systems are just a few sources to mention. In fact, once the concept is introduced many symplectic phenomena can be rephrased in a Lagrangian language, as a famous citation of A. Weinstein goes “Everything is Lagrangian”.

I will survey some developments in Lagrangian topology, which is a theory that mixes topological and symplectic invariants associated to Lagrangian submanifolds. I will explain how these can be applied to solve various problems in symplectic geometry. I will then move on to newer developments that have to do with the algebraic structure of the totality of these invariants, such as the Donaldson and Fukaya categories and their relations to Lagrangian cobordisms.

**Speaker:** Neset Deniz Turgay (Swansea)

**Title:** Conjugation Invariants in the Leibniz-Hopf algebra

**Abstract:** The Leibniz-Hopf algebra is the free associative algebra with one generator in each degree and co-product given by the Cartan formula. The mod 2 Steenrod algebra naturally occurs as a quotient of this Hopf algebra by the Adem relations. The ring of conjugation invariants in the dual Steenrod algebra arises when one considers commutativity of ring spectra. Motivated by this, Martin Crossley and I have studied the fixed points in the mod 2 dual Leibniz-Hopf algebra under this conjugation action.

We found that, like in the dual Steenrod algebra, these invariants are “approximately” half of the whole algebra, although we are able to give a much more precise statement than was possible for the Steenrod algebra. In this talk, I will show how to describe the conjugation invariants in the integral Leibniz-Hopf algebra, using the previous results in dual case.

**Speaker:** Alessandro Sisto (Oxford)

**Title:** Mapping tori of pseudo-Anosovs

**Abstract:** Pseudo-Anosovs are the most interesting self-homeomorphisms of surfaces. It is an important theorem of Thurston that when identifying the boundary components of  $S \times [0, 1]$  via a pseudo-Anosov, where  $S$  is a closed surface, the resulting 3-manifold admits a hyperbolic structure. I'll present an approach to this result based on geometric group theory and, time permitting, discuss other settings where the same techniques can be exploited. Based on joint work with Sebastian

Hensel.

## Saturday, Sept. 8

**Speaker:** Dorothy Buck (Imperial)

**Title:** The Topology of DNA–Protein Interactions.

**Abstract:** The central axis of the famous DNA double helix is often constrained or even circular. The topology of this axis can influence which proteins interact with the underlying DNA. Subsequently, in all cells there are proteins whose primary function is to change the DNA axis topology – for example converting a torus link into an unknot. Additionally, there are several protein families that change the axis topology as a by-product of their interaction with DNA.

This talk will describe some typical DNA conformations, and the families of proteins that change these conformations. I’ll present a few examples illustrating how 3-manifold topology has been useful in understanding certain DNA-protein interactions, and discuss the most common techniques used to attack these problems.

**Speaker:** Chris Wendl (UCL)

**Title:** Some tight contact manifolds are tighter than others

**Abstract:** In this talk, I will explain how overtwistedness and Giroux torsion can be understood as occupying the first two levels in an infinite hierarchy of filling obstructions, defined via the relationship between contact structures and open book decompositions. In particular, every contact 3-manifold has a numerical “degree of tightness” which is preserved under contact surgery and can be measured via Symplectic Field Theory (joint work with J. Latschev). If time permits, I will also explain some recent progress (joint with P. Massot and K. Niederkruger) toward understanding the corresponding hierarchy in higher dimensions.

**Speaker:** Zoltán Szabó (Princeton)

**Title:** Knot Floer homology, Kauffman states and bordered algebras

**Abstract:** Knot Floer homology is defined for knots in closed oriented three-manifolds and it is closely related to Heegaard Floer homology. The simplest version is a bigraded homology theory for knots in  $S^3$ . After reviewing some basic properties and open problems, we will study a new computational method that corresponds to a natural Heegaard Diagram for knot projections. This is a joint work with Peter Ozsváth.