Does Mathematics need a Philosophy? A paper for a TMS Symposium on 21/x/2013

Thomas Forster

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Let me start by confessing that my first degree was in Philosophy.

Is having a philosophy of mathematics something like having an idea of what we are supposed to be doing? A bit like having a project?

New wave capitalism has appropriated the word 'philosophy'. Every damned company has a philosophy: even drug dealers and payday lenders have a philosophy. I typed 'our corporate philosophy' into google and got 80 million hits. It's just a fancy way of saying what they think they are trying to do. Embodied in a mission statement. I have no objection to mission statements as long as they are in Latin.

Does mathematics have a philosophy in this sense? What is it that we think we are setting out to do when we embark on a day's work in mathematics? (Richard Whatsit's story about long divisions).

There are two pictures of what mathematics is. there is Abstract nonsense and romantic nonsense.

Romantic Nonsense

Romantic nonsense is the one i learnt at school. Need Intension-extension (a concept from philosophy) to explain it. Richard Watts-Tobin was a Selwyn JRF when my father was a fellow and i was sent to him for coaching. He tried to explain Rolle's theorem to me, but i was having none of it, not (at that stage) having the concept of arbitrary real-valued–function-in-extension. All farmers are poor; George is a farmer; Is George poor? There are cultures that cannot draw this inference: they say "Dunno ... who is this George?" C18th mathematics. Hardy said of Ramanujan that all the natural numbers were his personal friends. I saw 'Divergent Series' at school, and contemplating the series whose general term is $n! \cdot x^n$. There's romantic nonsense for you.

As late as 1963 textbooks were being written in which this point of view was set out with disarming honesty:

> "It seems to me that a worthwhile distinction can be drawn between two types of pure mathematics. The first—which unfortunately is somewhat out of style at present—centres attention on particular functions and theorems which are rich in meaning and history, like the gamma function and the prime number theorem, or on juicy individual facts like Euler's wonderful formula
>
> $$1 + 1/4 + 1/9 + \cdots = \pi^2/6$$
>
> The second is concerned primarily with form and structure."

George F Simmons, Topology and Modern analysis p ix.

Simmons' romantic nonsense view of Mathematics is Mathematics as the study of interesting intensions. Unfortunately the road to Hell is paved with interesting intensions.

This view of Mathematics is sometimes parodied by the other camp as *Mathematics as stamp collecting* or *Mathematics as butterfly collecting*; I rather like *Mathematics as egg-stealing*.

As you can tell, I am an abstract nonsense chap myself. But don't get me wrong. I like romantic nonsense as much as the next man. The paradigmatic piece of romantic nonsense is of course Number Theory. And don't we all love Number Theory?

Abstract Nonsense

It was in the 19th century that mathematics started to enunciate a mission that we now recognise, as the endeavour to generalise. TWK sez that's when mathematics became self-conscious. I don't know the earlier literature by mathematicians on what the purpose—the *mission*—of mathematics is.

This is what Mordell (or was it Siegel?) calls the theory of the empty set.

The Philosophy of Mathematics according to people of the abstract nonsense camp is the view that Mathematics is *anything done properly* or at least anything done with sufficient rigour. Anyone who is trying to formalise something new, so that they can reason about it reliably and expeditiously, is pushing out the frontiers of Mathematics. So we are always trying to annexe things to mathematics. And we have annexed quite a lot of mathematics in the last couple of hundred years. Most strikingly we have annexed a lot of metamathematics: Adrian says that a logician is someone who thinks that a formula is a mathematical object. I would go further: a logician will think that theories and interpretations between theories are mathematical objects. If that is what a logician is we should all be logicians. But it's not just metamathematics . . . there's graphs, knots

Mind you, some of the marches are not properly pacified: we still don't know the right way to think of proofs or games or knots as mathematical objects, and these are all pretty mainstream things.

To an abstract nonsense chap like me, the acceptance of the concept of arbitrary real-valued function in extension is as important—as *mathematically* important—a development as is the recent solution of the odd case of Goldbach's conjecture.

Most work on philosophy of mathematics is being done in Computer Science departments.

Ken Manders' remark, which i pass on (on the principle that it takes no more than five steps for a message to reach its destination) is that formalised versions of pre-mathematical objects always contain spurious detail.

How can official philosophers help with this project? You might think that philosophers are people who ask creative and annnoying questions instead of actually doing any work but i think they can help quite a lot.

And i don't just mean that some of the concepts we get from philosophy are important to working mathematician. Function-in-intension vs function-in-extension essential for computability.

Carve nature at the joints. "Essays in conceptual analysis" ...

Right tools in place when you need them. "What is the right way to think about this phenomenon?" is, according to the conception of philosophy that i was introduced to as a philosophy student, a philosophical question. It's the philosophy (in the sense of mission statement) of Analytic Philosophy.

Beautifully parodied by Frayn. Think of the house as an extension of the car rather than the other way round (The Tin Men).

If you are a philosopher, and you think that your job is what i've just been describing, then it's not surprising that you might think that philosophy is a portable skill. (Lots of people in management think that once you have a degree in management you can manage anything.)

Thus any discipline X spawns a discipline of Philosophy-of-X, so we have an operator that takes subject areas to subject areas. (Fortunately it is idempotent: it would be terrible if there were infinitely many subject areas for the humanities funding agencies to support.) But let that pass.

Unfortunately most of what passes for Philosophy of Mathematics does not arise from the praxis of Mathematics. In fact i even believe that the entirety of the activity of "Philosophy of Mathematics" as practiced in philosophy departments is—to a first approximation—a waste of time, at least from the point of view of the working mathematician.[1]. But There are various reasons why this might be the case. As Peter Smith says ...

In leaving the two concepts of abstract and romantic nonsense, i offer you the thought that Galois theory is the perfect combination of abstract nonsense and romantic nonsense. *Everybody* loves Galois theory.

Mention phlogiston theories at this point. Phlogiston is the wrong way to think about combustion. What is the status, therefore, of phlogiston theories? They're not entirely vacuous, beco's one can actually say true things with them ...can't one? I have tried to look at the philosophical literature on entities postulated by obsolete theories. [Philip Kitcher]. Cylindrical algebras the wrong way to think about algebrising first-order logic. Pointillism [Jeremy Buterfield's expression] is the wrong way to think about regions of space. [one thing on which Imre and I agree is that the oddity of Banach-Tarski is nothing to do with the axiom of choice, but is to do with thinking of regions of space as sets of points.]

[1]One should add that there are some philosophy departments—i know of two in Pittsburgh—that house people who know a great deal of mathematics and have useful things to say about mathematical praxis. But they are not engaged in the activity i mention in the scare quotes.