FOUR-MANIFOLDS **LENT 2009**

What's Examinable; What's Not

The exam is three hours long. There will be five questions; answer any three. Topics for the questions will be drawn from the following list:

1. Background on 4-manifolds
   (a) Intersection Forms; classification of indefinite quadratic forms.
   (b) Chern classes of algebraic curves and surfaces; the adjunction formula.
   (c) Topology of the K3 surface as a quartic surface in $\mathbb{CP}^3$; as an elliptic fibration; as the Kummer surface.
   (d) Handle decompositions and Kirby diagrams; the linking matrix, computing homology groups from a diagram.
2. Axiomatic properties of Floer homology. You should be able to give correct statements of these properties and apply them to prove other results.
   (a) Definition of the groups $\widehat{C}, C^+, C^-, C^\infty$. Exact sequences relating them.
   (b) Decomposition into $Spin^c$ structures. Given the intersection form on a four-manifold, be able to describe the set of $Spin^c$ structures in terms of c_1.
   (c) Functoriality.
   (d) The exact triangle (various forms.)
   (e) Absolute gradings.
   (f) Structure of HF^∞ when $b_1 = 0$; maps induced by cobordisms.
3. Heegaard Floer homology
   (a) Definition of the chain complex $CF(\Sigma, \alpha, \beta)$.
   (b) Find generators of $CF(\Sigma, \alpha, \beta)$ from a Heegaard diagram.
   (c) Divide generators into equivalence classes corresponding to $Spin^c$ structures.
   (d) Correspondence between maps $\phi : D^2 \to \mathrm{Sym}^g\,\Sigma$ and branched g-fold covers of D^2.
   (e) Simple examples of domains $\mathcal{D}(\phi)$ for which $\#\overline{\mathcal{M}}(\phi) = \pm 1$. (*e.g* disks and rectangles.) Why does the moduli space contain one point in these examples?

(f) Examples: the exact triangle for the unknot and trefoil knot. Twisted coefficients for $S^1 \times S^2$.

4. Applications. You should be able to give proofs of these starting from the axioms.

   (a) Donaldson's theorem. You can assume Elkies' theorem, but should know why it's true for even intersection forms.

   (b) Definition of $\Phi(M, \mathfrak{s})$ for $b_2^+(M) \geq 3$.

   (c) The adjunction inequality. (Starting from a correct statement about $HF(S^1 \times \Sigma_g)$.)

   (d) Lefshetz pencils; nonvanishing of $\Phi(X)$ for X an algebraic surfaces.

Some things which are NOT examinable:

1. Proof of structure of HF^∞ .

2. Knot Surgery.

3. Holomorphic triangles and definition of maps induced by cobordisms in Heegaard Floer homology.