IB GEOMETRY **LENT 2013**

Geometry of the Euclidean Plane

Lines

- A line is the shortest path between two points.
- Plane separation: the complement of a line is a disconnected topogical space.
- There is a unique line passing through two distinct points.
- Two distinct lines intersect in at most one point.
- Given a point $\mathbf{x}$ and a line L not containing $\mathbf{x}$, there is a unique line passing through $\mathbf{x}$ and parallel to L.
- Given a point $\mathbf{x}$ and a line L not containing $\mathbf{x}$, there is a unique line passing through $\mathbf{x}$ and perpendicular to L.

Circles

- A line and a circle intersect in at most two points.
- Two distinct circles intersect in at most two points.
- The perimeter of a circle of radius R is $2\pi R$.

Isometries

- If F_1 and F_2 are orthogonal frames, there is a unique isometry taking F_1 to F_2.
- An isometry which fixes three non-colinear points is the identity.
- Any isometry can be written as the composition of ≤ 3 reflections.

Triangles

- The sum of the interior angles in a triangle is π.
- If A_1, A_2, A_3 and A_1', A_2', A_3' are two sets of non-colinear points with $d(A_i, A_j) = d(A_i', A_j')$, then there is a unique $\phi \in \text{Isom}(\mathbb{R}^2)$ with $\phi(A_i) = A_i'$.
- If A_1, A_2, A_3 and A_1', A_2', A_3' are two sets of non-colinear points with $d(A_1, A_j) = d(A_1', A_j')$ and $\angle A_2 A_1 A_3 = \angle A_2' A_1' A_3'$ then there is a unique $\phi \in \text{Isom}(\mathbb{R}^2)$ with $\phi(A_i) = A_i'$.

Trigonometry

If $\triangle ABC$ has sides a, b, c and opposite angles α, β, γ, then

$$\frac{\sin\alpha}{a} = \frac{\sin\beta}{b} = \frac{\sin\gamma}{c} \qquad c^2 = a^2 + b^2 - 2ab\cos\gamma.$$

J.Rasmussen@dpmms.cam.ac.uk