RESEARCH SUMMARY

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Contents

My research interest is in the generalization of global/local class field theory using arithmetic geometry, in particular the Langlands correspondence and the arithmetic geometry of Shimura varieties. So far my work has been focused on the cohomological (algebro-geometric) realization of the representation-theoretic phenomena in this branch of algebraic number theory.

1. Weight spectral sequence and Hecke correspondences (2003-; in progress)

Let K be a complete discrete valuation field with a finite residue field k with $\operatorname{char} k = p > 0$. Let $\mathcal{O}_K$ be the integer ring of K, we fix a uniformizer π of $\mathcal{O}_K$, and let $k = \mathcal{O}_K/\pi \cong \mathbb{F}_q$. Let X be a strictly semistable scheme over $\mathcal{O}_K$ of relative dimension n, i.e. (1) X is proper and flat over $\mathcal{O}_K$ and X_K is smooth, (2) X is Zariski locally etale over $\mathcal{O}_K[[T_1, ..., T_{n+1}]]/(\pi - T_1 \cdots T_m)$ for some $1 \leq m \leq n$. For simplicity, assume that X_K is geometrically connected and the special fiber X_k is the union of $n+1$ irreducible components: $X_k = \bigcup_{i=1}^{n+1} Y_i$ with Y_i proper smooth and geometrically irreducible over k for $1 \leq i \leq n+1$.

T. Saito defines a strictly semistable resolution of the self-product $X \times_{\mathcal{O}_K} X$ in [SaT]. We denote this resolution by $f : X' \to X \times_{\mathcal{O}_K} X$. This f is an isomorphism on the generic fibers. If we let $Y_{i,j} = Y_i \times_k Y_j$ for $1 \leq i, j \leq n+1$, we have

$$(X \times_{\mathcal{O}_K} X)_K = X_K \times_K X_K, \quad (X \times_{\mathcal{O}_K} X)_k = \bigcup_{i,j} Y_{i,j}.$$

Let D_{ij} be the proper transform of $Y_{i,j}$ in D_{ij}. Then $X'_k = \bigcup_{i,j} D_{ij}$ and D_{ij} are the irreducible components of the special fiber of X'.

Now let $T_K \subset X_K \times X_K$ an irreducible closed subvariety of dimension n, regarded as an algebraic correspondence on X_K. Let T be the closure of T_K in X'. Denote the projection maps by p_1, p_2. Now we make the following basic assumption:

(1.1) The maps $f(T) \to X$ induced by p_1, p_2 are finite.

This assumption is satisfied in case of the application that we have in mind, namely the Hecke correspondences (at p) on certain Shimura varities. Under this assumption, we can make the following definition (we denote by $Z_i(S)$ (resp. $A_i(S)$) the abelian group of i-dimensional cycles (resp. rational equivalence classs of cycles) on S):

Definition 1.1. Let T_{ij} be the intersection cycle $T \cdot D_{ij}$, as the element of $Z_n(T \cap D_{ij}) \subset Z_n(D_{ij})$. Let $T_{i,j} = f_*(T_{ij}) \in Z_n(Y_{i,j})$.

Note that by our assumption these elements are well-defined cycles, not only rational equivalence classes of cycles.

Now for a subset $S \subset \{1, 2, ..., n+1\}$ of cardinality k, let $Y_S = \bigcap_{i \in S} Y_i$, which is a proper smooth variety over k of dimension $n - (k-1)$. For two subsets S, S', let $Y_{S,S'} = Y_S \times_k Y_{S'} \subset X_k \times_k X_k$, and when $|S| = |S'| = k$ and $S = \{s_1, ..., s_k\}$, $S' = \{s'_1, ..., s'_k\}$ with increasing order, define $D_{S,S'} = \bigcap_{i=1}^{k} D_{s_i, s'_i} \subset X'_k$, which is a proper smooth variety over k of dimension $2n - (k-1)$. Then, by elaborating Saito's construction, we prove:

Proposition 1.2. *$D_{S,S'}$ is generically a $(\mathbb{P}^1)^{k-1}$-bundle over $Y_{S,S'}$.*

We generalize Definition 1.1 as follows:

Definition 1.3. When $|S| = |S'| = k$, consider the intersection class $T \cdot D_{S,S'} \in A_{n-k+1}(D_{S,S'})$, and let $T_{S,S'} = f_*(T \cdot D_{S,S'}) \in A_{n-k+1}(Y_{S,S'})$.

Now under the previous assumption, we can show (inductively) that $T_{S,S'}$ is a well-defined $(n-k+1)$-dimensional cycle on $Y_{S,S'}$. The following theorem is proven using the projection formula:

Theorem 1.4. *Assume $|S| = |S'| + 1 = k + 1$. Then we have the equality :*

$$\Big(\sum_{i=1}^{k+1} (-1)^i \, T_{S-\{s_i\},S'}\Big) \cdot Y_{S,S'} = \sum_{j \notin S'} (-1)^{i(j)} \, T_{S,S' \cup \{j\}}$$

in $A_{n-k}(Y_{S,S'})$. Here $i(j) = |\{s \in S' \mid s < j\}| + 1$. Under our assumption, this is an equality in $Z_{n-k}(Y_{S,S'})$.

The cycles $(T_{S,S'})_{|S|=|S'|=k} \in Z_{n-k+1}(Y^{[n-k+1]})$, where $Y^{[n-k+1]} = \coprod_{|S|=k} Y_S$, can be seen as an algebraic correspondence on $Y^{[n-k+1]}$. Saito proves in [SaT] that the action of this correspondence on the ℓ-adic cohomology of $Y^{[n-k+1]}$ is compatible with the action of the correspondence T_K on the ℓ-adic cohomology of X_K under the weight spectral sequence. Thus, the cycles $T_{S,S'} \in Z_{n-k+1}(Y_{S,S'})$ computed via above theorem describes the action of T_K on the ℓ-adic cohomology of X_K via the weight spectral sequence.

We will apply this to a class of unitary Shimura varieties (including the case treated in [TY]) and actions of Hecke correspondence at p, to determine the representations of $GL_n(K)$ occurring in the weight spectral sequence of unitary Shimura varities with semistable reduction at p. This refines the computation in [TY] — we explain the context in the next section.

2. Compatibility of local and global Langlands correspondences (2003-04, [TY])

The non-abelianization of local class field theory, namely the local Langlands correspondence for general linear groups, was proven by Harris-Taylor [HT] (and Henniart [He]) in 1999. The method of Harris-Taylor was to first descend from an automorphic representation of general linear group over CM field to that of certain unitary group (essentially the multiplicative group of a division algebra) over the totally real subfield (due to Clozel), and then use the realization of the global Langlands correspondence in the ℓ-adic cohomology of unitary Shimura varieties (a refinement of results of Kottwitz). There they had shown that the global Langlands correspondence is compatible with the local Langlands correspondence, at all primes not dividing ℓ, at the level of Grothendieck groups (using the method of comparing trace formulae, which originated from Ihara and Langlands), but the compatibility including the monodromy operator was not known (see [H]). We settled this question in [TY]. The temperedness of the local component of the automorphic representation was already proven in Harris-Taylor, so the question is about the purity of the local Galois representation (the Weight-Monodromy Conjecture for the ℓ-adic cohomology of varieties over local fields in more general context). The main content of the paper [TY] is to show this purity of ℓ-adic cohomology for the appropriate unitary Shimura variety with semistable reduction, for the parts corresponding to the cuspidal automorphic representations. Now we state the precise result.

Let L be a number field (finite over $\mathbb{Q}$), n a positive integer, ℓ a fixed prime and $\iota : \overline{\mathbb{Q}}_\ell \cong \mathbb{C}$ a fixed field isomorphism. We denote the absolute Galois group of L by $G_L = \mathrm{Gal}(\overline{L}/L)$. Conjectural global Langlands correspondence predicts a correspondence between (A) algebraic automorphic representation Π of $GL_n(\mathbb{A}_L)$ and (B) n-dimensional ℓ-adic Galois representation $R : G_L \to GL_n(\overline{\mathbb{Q}}_\ell)$ which is (1) almost everywhere unramified and (2) de Rham at ℓ. The cuspidals in (A) should correspond to irreducibles in (B). We are interested in cuspidal Π, and we denote the (conjectural) Galois representation attached to Π by $R = R_{\ell,\iota}(\Pi)$. Up to semisimplification, it is characterized by the property that the eigenvalues of Frobenius Frob_v equal the Satake parameters of Π_v for almost all places of v. One of the most general results in the direction $\Pi \longmapsto R$ is the one obtained by Kottwitz [K], Clozel [C] and Harris-Taylor [HT], which constructs the semisimple representation $R_{\ell,\iota}(\Pi)$ when L is an imaginary CM-field and Π is cuspidal, satisfying (1) Π is conjugate self-dual, (2) Π is regular algebraic and (3) there is a finite place x of L where Π_x is (essentially) square-integrable. We show the compatibility of this correspondence with the local Langlands correspondence at all places outside ℓ; we need to fix notations for the local Langlands correspondence.

Let K be a finite extension of $\mathbb{Q}_p$ and n a positive integer. We denote the maximal unramified extension of K by K^{ur}, the (geometric) Frobenius by Frob, and the Weil group by $W_K = \{\sigma \in G_K \mid \sigma|_{K^{ur}} \in \mathrm{Frob}^{\mathbb{Z}}\}$. The local Langlands correspondence rec (proved by Harris-Taylor [HT] and Henniart [He]) gives the correspondence from (A$'$) irreducible admissible representations (over $\mathbb{C}$) of $GL_n(K)$ to (B$'$) n-dimensional F-semisimple Weil-Deligne representations (over $\mathbb{C}$) of W_K. Recall (see Tate [Ta]) that a Weil-Deligne representation is a pair (r, N) of a finite dimensional representation $r : W_K \to GL(V)$ and an $N \in \mathrm{End}(V)$ such that $r(\sigma)N = \chi(\sigma)Nr(\sigma)$ for all $\sigma \in W_K$, where $\chi : W_K \to \mathbb{Q}^\times$ is the composite of the local reciprocity map $W_K \to W_K^{ab} \cong K^\times$ (sending lifts of Frob to uniformizers) and the normalized absolute value $|\,|_K : K^\times \to \mathbb{Q}^\times$. We can define the F-semisimplification $r^{F\text{-ss}}$ of r, and write $(r, N)^{F\text{-ss}} = (r^{F\text{-ss}}, N)$ and $(r, N)^{\mathrm{ss}} = (r^{F\text{-ss}}, 0)$. The cuspidal (resp. square integrable, tempered) representations in (A$'$) correspond to irreducible (resp. indecomposable, pure) representations in (B′). For the definition of pure Weil-Deligne representations, see [TY]. For a prime ℓ and an ℓ-adic Galois representation ρ of G_K (assume de Rham when $\ell = p$), denote the associated Weil-Deligne representation (over $\overline{\mathbb{Q}}_\ell$) of W_K by $\mathrm{WD}(\rho)$ (for $\ell \neq p$ use quasi-unipotence of Grothendieck; for $\ell = p$ use Berger's theorem and Fontaine's functor D_{pst}).

Theorem 2.1. (Harris-Taylor [HT], Taylor-Yoshida [TY]) Let L be a CM-field, ℓ a prime and fix $\iota : \overline{\mathbb{Q}}_\ell \to \mathbb{C}$. Let Π be a cuspidal automorphic representation of $GL_n(\mathbb{A}_L)$ satisfying the three conditions above, and $R_{\ell,\iota}(\Pi)$ be the associated ℓ-adic representation of G_L. Then, for all finite place v of L not dividing ℓ:

$$\iota\mathrm{WD}(R_{\ell,\iota}(\Pi)|_{G_{L_v}})^{F\text{-ss}} \cong \mathrm{rec}\big(\Pi_v^\vee \cdot |\det|_K^{\frac{1-n}{2}}\big)$$

as Weil-Deligne representations over $\mathbb{C}$ of W_{L_v}.

Some remarks on the theorem:

(1) As we assumed the existence of x where Π_x is square-integrable, we obtain the irreducibility of $R_{\ell,\iota}(\Pi)$ if x does not divide ℓ. This is because $R_{\ell,\iota}(\Pi)$ is semisimple by definition, and the theorem tells us that its restriction at x is indecomposable.
(2) The equality of $(\,)^{\mathrm{ss}}$ of both sides in the theorem was one of the main results of Harris-Taylor ([HT], Introduction, Theorem C). The new result in [TY] is the determination of the monodromy operator N.
(3) In [TY], the theorem for v dividing ℓ is shown to follow from the functoriality of the p-adic weight spectral sequence of Mokrane [Mo].

We sketch the idea of proof of the theorem. First we note that the temperedness of Π_v is shown in [HT] (Introduction, Theorem C). Hence, by (Theorem A)$^{\mathrm{ss}}$, it suffices to prove that the left hand side is pure (in particular, this follows from the Weight-Monodromy Conjecture). Using the global base change, we reduce to the case when Π_v has a fixed vector by the Iwahori subgroup $\mathrm{Iw}_n = \{g \in GL_n(\mathcal{O}_{L,v}) \mid g \bmod v \text{ is upper triangular }\}$. We descend Π to an automorphic representation π of a unitary group G which locally at v looks like GL_n and at infinity looks like $U(1, n-1) \times U(0, n)^{[L:\mathbb{Q}]/2-1}$. Then we realise $R_{\ell,\iota}(\Pi)$ in the cohomology of a Shimura variety X associated to G with Iwahori level structure at v. More precisely (assume the infinitesimal character to be trivial for simplicity), the representation $R_{\ell,\iota}(\Pi)$ appears inside the semisimplification of the π^p-isotypic component of $H^{n-1}(X, \overline{\mathbb{Q}}_\ell)$. We show that X has strictly semistable reduction at v with a nice moduli-theoretic definition of the strata of the special fiber (the reduction of X at v), and use the results of [HT] to compute the cohomology of these (smooth, projective) strata as a virtual $G(\mathbb{A}^{\infty,p}) \times \mathrm{Frob}_v^{\mathbb{Z}}$-module. This description and the temperedness of Π_v shows that the π^p-isotypic component of the cohomology of any strata is concentrated in the middle degree. This implies that the π^p-isotypic component of the Rapoport-Zink weight spectral sequence ([RZ], [SaT]) degenerates at E_1, which shows that $\mathrm{WD}\big(H^{n-1}(X, \overline{\mathbb{Q}}_\ell)[\pi^p]|_{G_{L_v}}\big)$ is pure.

In the special case that Π_v is a twist of a Steinberg representation and Π_∞ has trivial infinitesimal character, the above theorem presumably follows from the results of Ito [I]. After we had posted the first version of this paper, Boyer [B] has announced an alternative proof with presumably stronger results.

As for the functoriality of the p-adic weight spectral sequence (cristalline/log-cristalline cohomology) alluded to in remark (3) above, we are investigating the question with T. Ito and Y. Mieda.

3. Non-abelian Lubin-Tate theory and Deligne-Lusztig theory (2002-03, [Yo3])

We retain the notation for the local Langlands correspondence in the previous section (in particular, K is a finite extension of $\mathbb{Q}_p$). As we sketched in the previous section, at present the proof of the local Langlands correspondence relies heavily on the partially established theory of the global Langlands correspondence, just as when the proof of the local class field theory was first deduced from the global class field theory. Recently the arithmetic geometry concerning the local Langlands correspondence is actively studied (see [H]). One important goal is the purely local proof of the so-called *non-abelian Lubin-Tate theory* formulated by Carayol [Car], which asserts that the local Langlands correspondence for $GL_n(K)$ is realized in the vanishing cycle cohomology of the deformation spaces of the formal $\mathcal{O}_K$-module with Drinfeld level structures, which are the local rings of the integral model of the Shimura variety used in the theory of Harris-Taylor (it is proven in the work of Harris-Taylor by a global method). Our result in this direction ([Yo3]) is a purely local proof of the non-abelian Lubin-Tate theory in a special case, in which we showed that it is geometrically equivalent to the theory of Deligne-Lusztig concerning the representations of reductive groups over finite fields.

Now we denote the integer ring of K by $\mathcal{O}_K$ and its residue field by $k \cong \mathbb{F}_q$. Fix a uniformizer π of $\mathcal{O}_K$ and a positive integer $n \geq 1$. Let X be the spectrum of the deformation ring of formal $\mathcal{O}_K$-module of height n with level π structure ([Dr]), which is a scheme of relative dimension $n-1$ over the integer ring $W = \widehat{\mathcal{O}_K^{\mathrm{ur}}}$ of the completed maximal unramified extension $\widehat{K^{\mathrm{ur}}}$ of K, which is a complete DVR with the residue field $\overline{k}$. We are interested in ℓ-adic etale cohomology groups $H^i(X_{\overline{\eta}}, \overline{\mathbb{Q}}_\ell)$ ($\ell \neq \mathrm{char}\, k$) of the geometric generic fiber $X_{\overline{\eta}} = X \times_{\mathrm{Spec}\, W} \mathrm{Spec}\, \overline{\widehat{K^{\mathrm{ur}}}}$, which are finite dimensional $GL_n(k) \times I_K$-modules, where I_K is the inertia group of K.

On the other hand, let DL be the Deligne-Lusztig variety over $\overline{k}$ for $GL_n(k)$, associated to the element $(1, \ldots, n)$ of the Weyl group of GL_n regarded as the symmetric group of n letters, or equivalently to a non-split torus T with $T(k) \cong k_n^\times$ where k_n is the extension of k of degree n ([DL]). As this variety has an action of $GL_n(k)$ and $T(k) \cong k_n^\times$, we can regard $H_c^i(DL, \overline{\mathbb{Q}}_\ell)$ as a $GL_n(k) \times I_K$-module by the canonical surjection $I_K \to k_n^\times$. We denote the alternating sums of these cohomology groups as follows:

$$H^*(X_{\overline{\eta}}) = \sum_i (-1)^i [H^i(X_{\overline{\eta}}, \overline{\mathbb{Q}}_\ell)], \quad H^*(DL) = \sum_i (-1)^i [H_c^i(DL, \overline{\mathbb{Q}}_\ell)]$$

which are regarded as elements of the Grothendieck group of $GL_n(k) \times I_K$-modules. Then our main theorem on the vanishing cycle cohomology groups of X can be stated as follows:

Theorem 3.1. (i) *We have the equality $H^*(X_{\overline{\eta}}) = H^*(DL)$.*

(ii) *Among the $H^i(X_{\overline{\eta}}, \overline{\mathbb{Q}}_\ell)$, cuspidal representations π of $GL_n(k)$ and generic inertia characters χ of I_K (here* generic *means χ does not factor through any $k_m^\times$ with $m \mid n,\ m < n$ via the norm map $k_n^\times \to k_m^\times$) occur only in $H^{n-1}(X_{\overline{\eta}}, \overline{\mathbb{Q}}_\ell)$, where they are coupled as $\bigoplus \pi_\chi \otimes \chi$ by the usual correspondence $\pi_\chi \otimes \mathrm{St} = \mathrm{Ind}_{T(k)}^{GL_n(k)} \chi$ where* St *is the Steinberg representation of $GL_n(k)$.*

This theorem gives the local proof of the non-abelian Lubin-Tate theory in this particular case of level π, because of the following. The character χ of I_K that factors through $I_K \to k_n^\times$ can be extended to a character $\tilde{\chi}$ of G_L, where L is the unramified extension of K of degree n (N.B. $I_K = I_L$). If χ is generic, $\mathrm{Ind}_{G_L}^{G_K} \tilde{\chi}$ is an irreducible representation of G_K. On the other hand, we can pull back the cuspidal representation π_χ by the natural surjection

$GL_n(\mathcal{O}_K) \longrightarrow GL_n(k)$, and then induce (compact modulo center induction) to $GL_n(K)$ to obtain a supercuspidal representation of $GL_n(K)$ which corresponds to $\mathrm{Ind}_{G_L}^{G_K}\tilde{\chi}$ via the local Langlands correspondence.

These results are proved by purely local arguments, constructing a suitable model of X and computing the cohomology of the geometric generic fiber in terms of that of the special fiber. In its course we obtain important informations on the X, as follows:

Theorem 3.2. (i) *The W-scheme X is isomorphic to*

$$\operatorname{Spec} W[[X_1, \ldots, X_n]]/(P(X_1, \ldots, X_n) - \pi),$$

where $P \in W[[X_1, \ldots, X_n]]$ is of the form:

$$(\text{unit}) \cdot \prod_{(a_i \bmod \pi)_i \in k^n \setminus \{\underline{0}\}} \big([a_1](X_1) +_{\tilde{\Sigma}} \cdots +_{\tilde{\Sigma}} [a_n](X_n)\big)$$

where $[a_i]$ and $+_{\tilde{\Sigma}}$ are the formal operations of a formal $\mathcal{O}_K$-module over $W[[X_1, \ldots, X_n]]$ obtained by lifting the universal formal $\mathcal{O}_K$-module over X.

(ii) *There exists a generalized semistable model Z_{st} of X over W, i.e. a proper W-morphism $Z_{st} \to X$ which is an isomorphism on the generic fibers and Z_{st} being generalized semistable. Here generalized semistable means that its complete local rings at all the closed points are isomorphic over W to*

$$W[[T_1, \ldots, T_n]]/(T_1^{e_1} \cdots T_d^{e_d} - \pi) \quad (d \leq n)$$

with integers e_i all prime to $\operatorname{char} k$.

(iii) *Over the tamely ramified extension $W_n = W(\pi^{1/(q^n-1)})$ of W, X has a model whose special fiber contains a smooth affine variety over $\overline{k}$ which is isomorphic to DL as schemes with right $GL_n(k) \times I_K$-action.*

Here we will point out a few possibilities of the further research in the coming years.

(1) The local models treated above give good integral models for the unitary Shimura varieties of Harris-Taylor type, and we expect cohomological applications (p-adic/mod p in particular).
(2) The reason why we see the Deligne-Lusztig variety is to be clarified by giving a suitable moduli-theoretic interpretation of these models. (The relation between the Drinfeld level structures of formal modules and the theory of Dieudonne crystals or displays associated to formal modules.)
(3) Generalization to higher level: this work should be extended to the deformation space with level π^m-structures for $m > 1$, eventually covering the whole of the local Langlands correspondence for GL_n. The case $m > 1$ is representation-theoretically quite different from the $m = 1$ case, and we expect it to have a simpler geometric interpretation. For this, the moduli theoretic interpretation should become important.
(4) Generalization to other groups: as Deligne-Lusztig theory treats more general reductive groups than GL_n, the corresponding representations of the groups over local fields should be realized in the cohomology groups of certain moduli spaces of formal groups, e.g. some variations of Rapoport-Zink spaces. This would be an important step towards establishing the local Langlands correspondences for the general reductive groups.
(5) ϵ-factor: the local Langlands correspondence for GL_n is formulated in terms of L-factors and ϵ-factors defined on both representation-theoretic and Galois-theoretic sides. We should clarify the coincidence of ϵ-factors via arithmetic geometry.

4. Class field theory of varieties over local fields (2001-02, [Yo1],[Yo2])

4.1. **Finiteness theorem in class field theory of varieties over local fields ([Yo1]).** In this paper we proved a finiteness theorem on the geometric part $\pi_1^{ab}(X)^{\mathrm{geo}}$ of the abelian étale fundamental group $\pi_1^{ab}(X)$ of proper smooth algebraic variety X over a local field K. This group $\pi_1^{ab}(X)^{\mathrm{geo}}$ (roughly) classifies the geometrically connected abelian étale coverings of X, and the finiteness theorem claims that (we denote the residue field of K by k, which is always assumed to be finite):

(1) The torsion part of $\pi_1^{ab}(X)^{\mathrm{geo}}$ is finite.
(2) The quotient of $\pi_1^{ab}(X)^{\mathrm{geo}}$ by its torsion part is a finite free module over the profinite completion of $\mathbb{Z}$.
(3) The rank of this free part is equal to the k-rank (the rank of the maximal k-split torus) of the special fiber of the Néron model of the Albanese variety of X.

In particular, if X has potentially good reduction $\pi_1^{ab}(X)^{\mathrm{geo}}$ is a finite group. This theorem had been considered by S. Bloch [B] and S. Saito [SaS] in the case when X is a curve. In this paper the method of Bloch is generalized to the case of bad reduction using the technique of monodromy filtration of abelian varieties over local fields, and also to the positive characteristic case using the recent results on p-divisible groups by de Jong [dJ]. This method can be applied to the higher dimensional case, giving the most general form of finiteness theorem as above.

As an application, the main theorem of the unramified class field theory of curves over local fields by Bloch and S. Saito is completely proven, in which the p-primary part in the positive characteristic case had remained unproven.

4.2. **Abelian étale coverings of curves over local fields and its application to modular curves ([Yo2]).** In this paper, the torsion part of $\pi_1^{ab}(X)^{\mathrm{geo}}$ in the curve case is investigated further. In particular, the size of the most difficult part $\pi_1^{ab}(X)^{\mathrm{geo}}_{\mathrm{ram}}$, which classifies the abelian étale covering which ramifies completely over the special fiber, is shown to be controlled by the geometric information of the jacobian variety J of X. The main theorem is stated as follows (we assume the existence of a K-rational point of X) :

(1) The prime-to-p part of the dual of the finite group $\pi_1^{ab}(X)^{\mathrm{geo}}_{\mathrm{ram}}$ injects into the component group of the special fiber of the Néron model of J.
(2) The p-primary part of $\pi_1^{ab}(X)^{\mathrm{geo}}_{\mathrm{ram}}$ vanishes in the case where the absolute ramification index of K does not exceed $p-1$, and X has semistable reduction.

It is proven by elaborating the considerations on the monodromy filtration of J in the preceding paper. The theory of finite group schemes by Raynaud [R] is used in the proof of 2.

This result enables us to determine the group $\pi_1^{ab}(X_0(p)/\mathbb{Q}_p)^{\mathrm{geo}}$ for the modular curve $X_0(p)$ (the coarse moduli space of elliptic curves with $\Gamma_0(p)$-strucutures). In particular, $\pi_1^{ab}(X_0(p)/\mathbb{Q}_p)^{\mathrm{geo}}_{\mathrm{ram}}$, which is the whole of the torsion part of $\pi_1^{ab}(X_0(p)/\mathbb{Q}_p)^{\mathrm{geo}}$, is isomorphic to the component group Φ of the Jacobian. This is considered as the local analogue of the Mazur's global result $\pi_1^{ab}(X_0(p)/\mathbb{Q})^{\mathrm{geo}} \cong \Phi$ ([Ma]).

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