Non-abelian Lubin-Tate Theory (M24)

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Level

Graduate level; non-examinable

Description

Local Langlands correspondence is a non-abelian version of the local class field theory, but unlike in the abelian case, we do not know how to prove it purely locally. Its geometric realization, so called non-abelian Lubin-Tate theory, is proven via global theory over number fields. Assuming only some familiarity with local fields and arithmetic geometry (theory of schemes), we will introduce the basic notions in the theory - local class field theory, Lubin-Tate theory, formal groups of higher height and its deformation spaces (Lubin-Tate spaces), the l-adic vanishing cycles, Weil-Deligne representations, smooth representations of $GL(n)$ over local fields, local Langlands correspondence - and state the main theorem. We would like to give as many proofs as possible of the basic background results like Lubin-Tate theory, Drinfeld's deformation theory, monodromy theorem of ℓ-adic Galois representations, etc. Then we will treat the cases where the main theorem can be proven locally (tamely ramified case), and time permitting, explain the global proof by Harris-Taylor via Shimura varieties (at least, we will take a look at the abelian case, namely the theory of complex multiplication).

Pre-requisite Mathematics

Some exposure to p-adic fields, Galois theory and commutative algebra. Preferable to know some scheme theory.

Literature

1. K. Iwasawa, "Local Class Field Theory", OUP.

2. Many important original articles, not necessarily easy to read; to be introduced in the first class.