

# Set Theory and Logic

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This course is the sequel to the Part II course with the title ‘Logic and Set Theory’. Although it will touch most of the themes of that Part II course, and will have something of the character of a course with a title like “A Twenty-four Lecture Graduate Course in Logic” it will concentrate on Set Theory more than other areas. Topics likely to be covered by other logic-related courses in Part III will be eschewed. At least one topic that has recently been dropped from Part II—namely Recursive Function Theory—will be given an “advanced beginner” treatment. Other topics likely to be covered include: model theory background (ultraproducts, elementary embeddings, categoricity, saturation, Ehrenfeucht-Mostowski theorem); inner models: the consistency of ZF relative to intuitionistic ZF, (relative) consistency and independence of most of the axioms of ZFC. The constructible universe. Forcing. Infinitary combinatorics: the Erdős-Rado theorem on uncountable monochromatic sets, leading to measurable cardinals and other large cardinals; Axiom of Determinacy; Set theory in Analysis (e.g. Souslin Hypothesis); Fränkel-Mostowski models; Alternative set theories (very briefly); Borel determinacy; WQO and BQO theory at least as far as Kruskal’s theorem on wellquasiorderings of trees and possibly connections with Reverse Mathematics.

This is a *mature* course in the sense that I have been giving it for a number of years, with the result that the lecture notes have thoroughly cleaned up and can be made available to students with fewer health warnings than usual. The notes are linked from my home page (look for **teaching**) but are browsable only from .cam.ac.uk addresses. If you reading this from outside Cambridge and want a copy, let me know. The notes are also designed to be sufficiently full for students to be content with annotating them with their own comments rather than take full notes of their own from scratch: I shall lecture this course on the assumption that people attending it have access to the notes.

The notes are maximal in the sense that they cover all the material I might lecture rather than what I will in fact lecture—this last being to a certain extent under the control of the audience. In particular the coverage given to topics appearing in the Part III essays that I sponsor will depend on whether or not anyone is doing any of those essays. Those essays include “Alternative set theories” (which covers a multitude of sins and has been a popular choice) and WQO and BQO theory. My home page contains extensive notes of WQO and BQO theory—far more than I can get round to lecturing.

## Pre-requisite Mathematics

The obvious prerequisite is Professor Johnstone’s Part II Logic and Set Theory, and I am going to assume that everybody coming to my lectures is on top of all the material lectured in that course. If you are planning to come to my course of lectures I would strongly advise you to dig out your example sheets from Prof. Johnstone’s part II course and this time attempt **all** the questions.

If you are a student from outside Cambridge and did not attend the part II course contact the department and we will be able to supply you with the .pdf of the exercises. Additionally supervisions will be provided for students who did not attend the course in the first place and for any others who feel—for whatever reason—that they need help.

## Literature

These two books below both started life as lecture notes for the Part II Logic and Set Theory course (PTJ and I have both lectured this syllabus), and both cover the background, albeit in completely different ways—complementing each other admirably.

P. T. Johnstone: Notes on Set theory CUP

T. E. Forster: Logic, Induction and Sets CUP.

(Errata are on <http://www.dpmms.cam.ac.uk/~tf/typoslis.html>)

Teaching materials will be linked from the page on <http://www.dpmms.cam.ac.uk/~tf/partiii.html>