

This requires a critique of critique that will build a new trust in the political capacity of images. Images can contribute by drawing a new configuration of the sensible, but on the condition that they don't work for a political effect.

With this we arrive at the conclusion of Rancière's argument. In a chain of affirmative examples, Rancière analyses what he calls the 'pensive image' – title of the last essay. Cutting across a selection of twentieth-century photography and cinema, from Walker Evans and Lewis Payne to Abbas Kiarostami and Rineke Dijkstra, the pensive image encapsulates the principle that Rancière defends in his concept of the aesthetic regime: undecidability or indetermination due to the suspension of any attribution to the work of social or political origin, intention or destination. A pensive image hides a thought which affects the spectator without allowing her to attribute it either to the author of the image or to the subject of the portrait in the image. This is not a given condition for any artwork after 1800 – the modernist status of autonomy – but a result of sophisticated crossings between heterogeneous regimes of expression, which 'create new figures, awakening the sensible possibilities that have been exhausted'.

Although Rancière's account of contemporary medi-ality as relocation of the effect of one medium into another is strikingly acute, he doesn't theorize like a curator, who might go so far as to baptize a distinct aesthetic on the basis of a particular artistic operation. His agenda, and the performative success of these essays, is to disarm such curatorial debates. The question is who is destined for what message. *Le Spectateur émancipé* recommends that the artist 'experiment more', and the curator 'speculate less'; look into the possibilities of recasting the sensible, Rancière suggests, and you will find them abundant. But this precisely indicates the limit of Rancière's plea for emancipation. It remains bound to an analysis of representation in the form of the sensible without accounting for those registers of art's operations that concern the political economy of art production, as well as experiments with the forms of labour and sociality through which art might challenge the part in which it has been cast within society. However, this might well be to demand more than that which *Le Spectateur émancipé* actually aims for. For, in the end, what we are given is, above all, a figure of the spectator whose capacities to sense and think are greater than we have – since Lacan, Debord, Irigaray and other French 'denigrators' of the spectacle – been prepared to conceive.

**Bojana Cvejic**

## Number theory

Alain Badiou, *Number and Numbers*, trans. Robin Mackay, Polity Press, Cambridge, 2008. 240 pp., £16.99 pb., 978 0 7456 3879 9.

One of the more astonishing aspects of Alain Badiou's philosophical position is that the key to what is most distinctive about it can be summarized in just three words: mathematics is ontology. His major work, *Being and Event*, kicks off with this stark assertion, and proceeds to derive a series of bold conclusions – the wresting of ontology from Heidegger's embrace, the construction of a rigorous and rationalist metaphysics, and a wholesale refoundation of the relationship between philosophy and science, the latter henceforth being conceived as one of philosophy's 'conditions'.

It should be noted, however, that while Badiou identifies ontology with mathematics in its most general sense, *Being and Event* by and large concerns itself with a very specific field of mathematics, namely set theory. Badiou recasts this as the theory of 'pure multiplicity', a reference to the fact that sets do nothing more than gather together their multiple elements and count them as one. There are reasons for this choice, of course, not least of which is the role that set theory plays within mathematics. Set theory acts as a kind of internal ontology of mathematics, certainly in the weak sense that any mathematical entity can be thought of as a kind of set, and arguably in the strong sense that mathematical entities actually *are* sets. For example, the mathematical concept of an ordered pair  $\langle a, b \rangle$  is distinct from that of the set  $\{a, b\}$ . The former has an ordering that makes  $a$  its first element and  $b$  its second. The latter, in contrast, is a pure multiple without any kind of order inscribed upon it. But although ordered pairs are conceptually distinct from sets, they can be implemented as sets by defining the ordered pair  $\langle a, b \rangle$  as the set  $\{\{a\}, \{a, b\}\}$ . The reader can check that given any set of this form, one can extract the first and second elements from it. Ordered pairs can thus be simulated through the intricate weaving together of pure multiplicities. The same, arguably, is true of any other entity used in mathematics.

But while set theory plays an important foundational role in mathematics, that is almost all it does. The concepts and techniques it deploys are of little interest to the 'working mathematician', most of whom get by with only a smattering of knowledge of the field. Only occasionally does a problem in general mathematics turn out to revolve around set-theoretic considerations – though such occasions can and do arise, which is

why set theory cannot simply be dismissed a province for pedants and philosophers.

All this opens up an intriguing problem: what is the ontological significance of the rest of mathematics, the overwhelming bulk of mathematics, once one moves beyond the limited terrain of pure set theory? Far from being the final word on the question of being, Badiou's identification of mathematics with ontology opens the door to a vast 'meta-ontological' research programme, one that scours the entirety of contemporary mathematical thought, elucidating its concepts and thinking through their metaphysical implications. Indeed, Badiou's own work occasionally hints at this larger research programme. In his essay 'Group, Category, Subject', he argues that the mathematical theory of groups can act as a grounding framework for the psychoanalytic notions of subjectivity found in Lacan and Freud. In 'One, Multiple, Multiplicities', his rejoinder to Deleuzian critics, Badiou argues that notions of the 'open' and the 'closed' should ultimately refer back to the way these concepts are deployed in topology.

The most systematic exploration of a region of mathematics outside its foundational core comes in Badiou's short book *Number and Numbers*. It was published in 1990, a couple of years after *Being and Event*, and has now been expertly translated into English by Robin Mackay. In it Badiou examines what mathematicians call the 'surreal numbers' – a class of number-like entities that incorporate familiar species of number, such as the integers, the rationals and the reals, but also encompass less familiar ones such as transfinite ordinals and infinitesimals (i.e. infinitely small quantities). The surreal numbers were introduced by the mathematician John Horton Conway as a by-product of his investigations into Go, the ancient Japanese board game. Conway simply called his creations Numbers – the term 'surreal numbers' was coined by Donald Knuth in his peculiar 1974 booklet of the same name, the text that introduced Conway's creations to the wider public. Knuth's terminology has since stuck. Significantly Badiou reverts to calling them Numbers, despite the fact that in other respects his approach is diametrically opposed to Conway's recursive and constructivist presentation.

Badiou sets out his stall in the polemical opening pages of *Number and Numbers* – a chapter numbered zero and entitled "Number must be thought". In it he notes the profusion of different types of numbers, both within mathematics and in culture at large, and contrasts this empirical extravagance with the stubborn absence of any unifying *concept* of number. It is to remedy this deficiency that Badiou turns to the surreals

and presses them into service. He notes that the class of surreals subsumes all the heterogeneous entities we ordinarily like to think of as numbers, and a whole lot more besides. Yet as a class they can be defined in a uniform and relatively straightforward manner. They are both comprehensive and simple – and for Badiou the simultaneous presence of these two virtues is the calling card of the properly ontological. The surreal numbers are thus more than a curiosity or a neat trick: they capture the essence of number itself. The Numbers tell us what number *is*.

Much attention has been paid to the political gloss Badiou puts on his project here. The book's back-cover blurb presents his attempt to construct a rigorous concept of Number as a broadside against 'the political regime of global capitalism' and its reliance on a concept-less and ramified numerosity. Despite my sympathies with Badiou's leftist politics, I find this claim overblown. While it is certainly true that capitalism presses numbers into its ideological service, it is not clear how a rigorous concept of Number would per se challenge such abuses. And surely the problem with opinion polls, stock-market prices, econometric models and so on resides not in the maths as such, but in their tenuous relationship to reality. The now-discredited formulae used to price financial derivatives are still perfectly effective and compelling when used by physicists to model Brownian motion.

These caveats aside, Badiou is right to point out that contemporary thought has a blind spot when it comes to number, and right to attempt to remedy this deficiency. The next half-dozen chapters proceed to survey earlier attempts to think number by Frege, Dedekind, Peano and Cantor in the late nineteenth and early twentieth centuries. This is the most accessible section of the book and is valuable in its own right as a thorough introduction for non-specialists to the philosophical and mathematical issues at work here.

The treatment of Frege gives an insight into Badiou's approach. We start with a firm focus on the metaphysical stakes of Frege's project – the conviction that numbers can be engendered from pure thought. We are then guided through Frege's construction of number, its demolition and partial repair at the hands of Russell and Zermelo, before coming to Badiou's materialist critique. Frege ultimately fails because one cannot derive the existence of objects from pure thought. The existence of something rather than nothing is an ontological axiom, not a logical necessity. Yet there is a twist in the tail – Frege's masterstroke of starting his consideration of number from zero rather than one turns out to lay the foundations for a materialist

ontology capable of providing a framework for the thinking of Number. All this is achieved in nineteen terse, numbered paragraphs.

Having completed his historical survey, Badiou moves on to recapitulate certain aspects of set theory and ontology – material that will be familiar to those who have read *Being and Event* and that acts as a useful companion to that work. He then proceeds to use this set-theoretic machinery to define Numbers, demonstrate that they have a natural linear order, and prove a variety of theorems about them. The book culminates in the definition of basic arithmetical operations such as addition and multiplication, and the verification that these operations obey the standard algebraic laws one would expect. As is often the case in Badiou's work, the mathematics he presents is standard, though the presentation of it is tweaked to reflect his philosophical agenda. For instance, Badiou defines a Number to be a specified subset of a specified ordinal. This is not a standard definition, though it can be shown to be equivalent to those found in mathematical literature.

The merit of Badiou's approach here is its low ontological overhead. Number is defined more or less directly in terms of the basic set-theoretical relationships of belonging and inclusion. In particular, the definition goes through without reference to any prior notion of order, seriality or counting. Number is thus sundered from any kind of intuition or empiricism and rendered purely as a 'form of Being'. It is also worth noting that Badiou's approach to Numbers makes them appear 'all at once', so to speak. The entire field of surreal numbers is defined in one fell swoop – the weirdest and wildest Numbers born simultaneously and alongside familiar entities such as 2, -17 and  $\frac{1}{4}$ . This is in sharp contrast to Conway's generative approach that starts from the integers and progressively creates ever more complex surreals. The contrast is even sharper with Knuth's take on Conway, which is framed in explicitly theological terms as a creation parable involving God and a pair of maths-besotted hippies.

These and other fascinating technical intricacies aside, the big question is whether any of this works. Does Badiou supply a coherent, unifying concept of number that is consistent with his wider ontological project? Does he manage to succeed where others have failed in 'thinking Number'? In my judgement the answer is a provisional and cautious 'yes'. Badiou's metaphysical take on the surreals is bold and startling, but it does provide an answer to the question 'what is number?', albeit one that is most persuasive to those already partial to Badiou's views on these matters.

Nevertheless, some warnings are in order, most of which revolve around the mathematics of surreal numbers. Despite the astonishing beauty of the surreals, attempts to make use of them in wider mathematics have so far foundered (at least so far as I am aware). For instance, while the surreals admit particularly neat definitions of addition and multiplication, exponentiation proves to be significantly more awkward. Moreover, these definitions do not easily yield a practical algorithm for calculating arithmetic sums and products, as one might have hoped. And while the surreals include all manner of infinitesimal quantities, it has proved exceptionally difficult to develop calculus using these infinitesimals. The surreals promise much, but have so far delivered little.

But is it just a coincidence that the surreal numbers, like set theory, turn out to be of little practical use for the working mathematician? Perhaps there is a necessary disjunction between ontological importance and practical utility. Perhaps the 'use' of these regions of mathematics is precisely to act as an ontological foundation for the rest of mathematics, and we shouldn't expect anything more of them. Perhaps ontology is the discourse that picks up precisely at the point where practicality has nothing left to say. This would be an surprisingly Heideggerian conclusion to draw from Badiou's austere rationalist vision, but one that would be in keeping with his distrust of the dimly empirical.

**Anindya Bhattacharyya**

## Neither last nor least

Edward Skidelsky, *Ernst Cassirer: The Last Philosopher of Culture*, Princeton University Press, Princeton and Oxford, 2008. 288 pp, £19.95 hb., 978 0 6911 3134 4.

Following his death in 1945, Ernst Cassirer tended to be viewed in the anglophone world as a formidable and erudite intellectual historian with little of substance in terms of his own philosophical position, while in the German-speaking world he was seen as the most significant of the last generation of Marburg Neo-Kantians. Cassirer's magnum opus, the three-volume *Philosophy of Symbolic Forms* (1923–29), came to be viewed as the final defence of German idealism before Heidegger dealt it the final blow. This milestone was marked by the famous Davos encounter of March 1929, in which a young and ascendant existentialist