

On 5 and 7 Descents for Elliptic Curves

Thomas Anthony Fisher

Summary

We perform descent calculations for the families of elliptic curves over $\mathbf{Q}$ with a rational point of order $n = 5$ or 7 . These calculations give an estimate for the Mordell-Weil rank which we relate to the parity conjecture. We exhibit explicit elements of the Tate-Shafarevich group of order 5 and 7, and show that the 5-torsion of the Tate-Shafarevich group of an elliptic curve over $\mathbf{Q}$ may become arbitrarily large.

In a special case, namely when the 5-torsion of our elliptic curve splits as $\mu_5 \oplus \mathbf{Z}/5\mathbf{Z}$, we improve our estimate for the Mordell-Weil rank by using the Cassels-Tate pairing to perform a full 5-descent. We generalise our results to curves over $\mathbf{Q}(\mu_n)$ and finally make some calculations for the curve $X_1(11)$ over its field of 5-division points.