P.M.H. Wilson, Curved Spaces

Changes requested in any reprinting.

page 2, line 6 : $\|x\|^2\,\|y\|^2$ should be $\|\mathbf{x}\|^2\,\|\mathbf{y}\|^2$.

page 2, line 11 : $\|x\|^2 + 2(\mathbf{x},\mathbf{y})\lambda + \|y\|^2$ should be $\|\mathbf{x}\|^2 + 2(\mathbf{x},\mathbf{y})\lambda + \|\mathbf{y}\|^2$.

page 2, line -9 : $|\lambda+1|\,\|x\| = (|\lambda|+1)\,\|x\|$ should be $|\lambda+1|\,\|\mathbf{x}\| = (|\lambda|+1)\,\|\mathbf{x}\|$.

page 4, line -9 : Replace final sentence of Section 1.1 by:

A similar argument shows that the same is true for any small enough open neighbourhood of $\bar{P}$, and hence no open neighbourhood of $\bar{P}$ in X can be homeomorphic to an open disc in $\mathbf{R}^2$.

page 6 : Omit the first of the two end of proof signs on this page.

page 7, line 7 : $R(\mathbf{x}) = \mathbf{x}$ should be $R_H(\mathbf{x}) = \mathbf{x}$.

page 7 : Replace sentence after diagram (lines 10-12) by:

For any $\mathbf{a}, \mathbf{a}' \in H$ and $t, t' \in \mathbf{R}$, we observe that $(\mathbf{a}-\mathbf{a}')\cdot\mathbf{u} = 0$, and therefore

$$\begin{aligned}\|(\mathbf{a}+t\mathbf{u}) - (\mathbf{a}'+t'\mathbf{u})\|^2 &= \|(\mathbf{a}-\mathbf{a}') + (t-t')\mathbf{u}\|^2 = \|(\mathbf{a}-\mathbf{a}') - (t-t')\mathbf{u}\|^2 \\ &= \|(\mathbf{a}-t\mathbf{u}) - (\mathbf{a}'-t'\mathbf{u})\|^2 = \|R_H(\mathbf{a}+t\mathbf{u}) - R_H(\mathbf{a}'+t'\mathbf{u})\|^2;\end{aligned}$$

hence R_H is an isometry.

page 8, line 8 : Replace "We observe that .... ; moreover ...." by

We observe that $(\mathbf{p}-\mathbf{q})/2$ is normal to H; moreover ....

page 13, lines 7-9 : Delete the sentence "This property fails for example .... has infinite length.".

page 14, line -9 : In second displayed equation of proof, change:

$\xi \in (s,t)$.

to

$\xi \in [s,t]$.

page 14, lines -8 to -7 : Replace:

Therefore, if $|t - s| < \delta$, then

$$\|\Gamma(t) - \Gamma(s) - (t - s)\Gamma'(\xi)\| < \varepsilon(t - s) \qquad \text{for all } \xi \in (s, t).$$

by

Therefore, if $0 < t - s < \delta$, then

$$\|\Gamma(t) - \Gamma(s) - (t - s)\Gamma'(s)\| < \varepsilon(t - s).$$

page 22, Exercise 1.5 : Replace:

orbit of the origin under G, or otherwise,
by
orbit of the origin under G and using Theorem 1.5, or otherwise,

page 27, line 3 : Replace line by:

Noting that the non-reflex angle between $\mathbf{n}_2$ and $\mathbf{n}_3$ is $\pi - \alpha$,

page 27, line 9 : $|\mathbf{C}| = 1$ should be $\|\mathbf{C}\| = 1$.

page 29, line 8 : The displayed formula should be:

$$\sin\alpha\,\sin\beta\,\cos c = \cos\gamma\,+\,\cos\alpha\,\cos\beta.$$

page 29, line -7 : Definition 1.10 should be Definition 1.9.

page 31, line -4 : The first) after $\mathbf{y}/\|\mathbf{y}\|$ should be deleted.

page 32, line 15 : Replace:

has a fixed point in $\mathbf{R}^3$, namely
by
has a fixed point in $\mathbf{R}^3$ (Exercise 1.5), namely

page 42, first line of proof of Theorem 2.19: Replace:

The rotation $r(z, \theta)$ about the z-axis $\mathbf{R}(0, 0, 1)^t$, through an angle θ (clockwise), corresponds ....

by

The rotation $r(z,\theta)$ about the z-axis, through a clockwise angle θ about its positive generator $(0,0,1)^t$, corresponds ....

page 55 : Add an end of proof sign at the end of the assertion in Lemma 3.5.

page 82, line 2 : The right $\mathbf{u}_1$ in $(d\sigma)_{\pi(P)}\mathbf{u}_1 \cdot (d\sigma)_{\pi(P)}\mathbf{u}_1$ should be a $\mathbf{u}_2$.

page 85, line -10 : "Riemmanian" should read "Riemannian".

page 88, Exercise 4.6 : add to the end of exercise:

[Hint. To prove that an isometry does not exist, show that in one space there are curves of finite length going out to the boundary, whilst in the other space no such curves exist. This may be reinterpreted in terms of the corresponding metric spaces: one space is incomplete whilst the other space is complete.]

page 88, Exercise 4.8 : Consider $\mathbf{R}^2 \setminus \{uv = 0\}$ equipped ...

page 98 : Displayed equation should read:

$$\begin{aligned} 2\pi \int_0^{\tanh \frac{1}{2}\rho} 4r dr/(1-r^2)^2 &= 4\pi(1-\tanh^2(\rho/2))^{-1} - 4\pi \\ &= 4\pi(\cosh^2(\rho/2) - 1) = 2\pi(\cosh\rho - 1). \end{aligned}$$

page 98, end of Section 5.3 : Replace:

are also hyperbolic circles.

by

are also hyperbolic circles, since a Euclidean circle with centre $i\,c$ and radius $r < c$ is a hyperbolic circle with hyperbolic centre $i\,\sqrt{c^2 - r^2}$ and hyperbolic radius $\sinh^{-1}(r/\sqrt{c^2-r^2})$.

page 106, figure at bottom of page : Delete the label $a + r$ to the left of the vertical line (but keep the label $a + r$ bottom right).

page 107, line -1: Replace:

$$u^2 + v^2 = \frac{1-z^2}{(1+z)^2} = \frac{1-z}{1+z}$$

by

$$u^2 + v^2 = \frac{z^2 - 1}{(1+z)^2} = \frac{z-1}{z+1}$$

page 108, line 1: $r = u^2 + v^2$ should be $r^2 = u^2 + v^2$.

page 115, line -7: $\sigma_u(P) = d\sigma_p(e_1)$ and $\sigma_v(P) = d\sigma_p(e_2)$ **should be** $\sigma_u(P) = d\sigma_P(e_1)$ and $\sigma_v(P) = d\sigma_P(e_2)$

page 131, Exercise 6.11 Replace sentence:

By considering coordinates .....

by

Let U denote the open subset of the upper half-plane model of the hyperbolic plane given by $y > 1$; show that there is a smooth surjective map from U onto S given by $u = -\log y$ and $v = x$, which locally is an isometry.

page 134, line 18: Replace:

the integral at τ may be written,

by

the change to the integral for small τ may be written,

page 134, line 22: Replace:

we deduce that the integral may be written

by

we deduce that the change to the integral may be written

page 134, line -2: In the second integral on this line,

$$\frac{\partial v}{\partial t}$$

should be

$$\frac{\partial v}{\partial \tau}$$

page 139, line 10: Replace:

... such that $\mathrm{Im}\,\Gamma_1$ is contained in ....

by

... such that the image of Γ_1 is contained in ....

page 145, line 9: Delete the "*of*" in "*an open neighbourhood of* W".

page 150, line 4: Proposition 2.16 should be Theorem 2.16.

page 156, line 1-2 : Replace:

and the ϕ_{ij} *transition functions*

by

and the ϕ_{ij} are called *transition functions* or *coordinate transformations*

page 160, line 12-13: Replace:

B, C are distinct points .... joining B to C lies in $W \setminus \{A\}$.

by

B, C are distinct points of $W \setminus \{A\}$ which do not lie on a geodesic ray through A, such that the curve Γ of absolute minimum length joining B to C lies in W.

page 172, line -5 :

... one illustrated above, the surface of which is a 'rectangular torus', homeomorphic to a smooth torus. ...

page 174, line 13 : Replace:

are remarkably simple.

by

are natural generalizations of Proposition 3.13.

page 184 : In index entry 'metric, British Rail, 3, 13', delete reference to page 13.