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# Why do we study geometry?
# Answers through the ages

*An expanded version of the lecture given by*
*Piers Bursill-Hall*
*as*

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### *A simple answer*

Let me begin with the question that is the title of this talk: why should we study geometry, and so, in some distant sort of way, why should we build buildings such as the Centre for Mathematical Sciences and more importantly, the *Faulkes Institute for Geometry* that we celebrate today? Not the answer an educationalist or an enthusiastic pure mathematician might give today (and which Dr. Faulkes must have heard *ad nauseam* over the last few years!), but the kinds of reasons that have been given over the last two and a half thousand years, and which have been at the foundations of our scientific thinking since the ancient Greeks, and the story which our benefactors now consciously continue.

At least at one level, the answer is quite surprisingly simple. Over most of the last two and a half thousand years in the European or Western tradition, geometry has been studied because it has been held to be the most exquisite, perfect, paradigmatic truth available to us outside divine revelation. It is the surest, clearest way of thinking available to us.[1] Studying geometry reveals – in some way – the deepest true essence of the physical world. And teaching geometry trains the mind in clear and rigorous thinking.

What I mean here is a little more complicated than it might first appear; it is easy for us to justify the study of geometry *to ourselves* (although perhaps not so easy to the man on the top of the Clapham Omnibus), and our reasons today are very deeply embedded in and reflect the science and scientific culture of late 20th century Europe. Yet European culture has valued and studied geometry (and our benefactors have had their predecessors) over the last two and a half thousand years, reaching back to such institutions as Plato's *Academy* in the Athens of the turn of the 4th century BC. What is this continuing need or this continuing stimulus to study geometry? What is interesting is that the answers of the ancient Greeks were certainly not those we would give today. What has been the perceived value of studying, doing research into geometry? And – let's not be mathematicians for a moment – why on earth should something so arcane and generally useless as geometry (done the way mathematicians do geometry) be so apparently important, and continuously important to Western learning and values over two and a half millennia? Clearly there is a history here: a history of the role, status, the reasons past thinkers have had for the importance of geometry. And it might come as a surprise to some that past actors did not share the same scientific or intellectual values that we have today, and did not have anything like the same reasons for studying geometry as we have today.

If justifications of why we should study geometry change, the fascination with geometry seems to have been pretty much constant whenever there have been those who have enquired into the nature of the causes and functioning of the

[1] Indeed, its relationship with divine revelation is of more than passing importance, and this is one of the reasons why this simple answer is really so complex and profound in its ramifications.

world around us. Indeed, a concern with shapes and the regularities of things like lines, circles, triangles and squares seems to be one of the things that comes to human beings quite naturally, alongside such things as language, sex, counting, finding better ways to kill each other, shouting at football matches, and driving the M25 at 150 kph. Most (although not all) early more-or-less primitive societies and early civilisations had some practical knowledge about geometrical objects. The amount of careful geometry and geometrical astronomy needed to build a place like Stonehenge or the architecture of complex palace compounds in early Egypt, or China, or the calendars of the Mayans and Aztecs, and so on through most other early societies, shows that some quite detailed and precise practical knowledge of geometrical objects has been widespread through most civilised human societies.

However, such practical if precise knowledge is hardly any kind of scientific study of *geometry* in anything like the sense that we mean today. It is not apparently distinguished from any other kind of knowledge, and makes no claim to any sort of special or unique certainty: it is not *epistemically differentiated*, as we say in the trade. Put simply, it would seem that these early societies had no concept of knowing about geometry in a way that is different from knowing about physical things. Early civilisations may have had elaborate practical knowledge of geometrical facts, it was some sort of practical calculating art, without any notion that one could know about geometry in a different, special way. There wasn't anything particularly special about geometry or knowledge of geometrical things. So, beyond its utilitarian role, there was no particular or special reason to study geometry *qua* geometry.

## *Ancient Greeks*

This changed with the ancient Greeks, sometime late in the 6$^{th}$ or early in the 5$^{th}$ century BC. It was amongst the couple of generations around the turn of the century that the Greeks began to distinguish explanations of the world based on natural causes as opposed to explanations that involve supernatural causes. What particularly stuck them was that it was a lot easier to criticise and to justify explanations in terms of natural causes – one can subject such explanations to *reason* – reasoned criticism, reasoned argument and reasoned justification. Thus explanations of the phenomena of the world based on natural causes are open to a great deal more certainty and reliability than explanations of phenomena that invoke causes stemming from extra-natural or supernatural origins. This distinction – which is *not* the separation of science and religion, by the way – between a causal mechanics of phenomena based on natural and therefore knowable causes and supernatural and therefore less reliably knowable causes, is one of the great intellectual step that separated the thinking of the ancient Greeks from all their contemporaries (with the exception of the Chinese), and it is here that the tradition of western rationalist thinking begins, or philosophy and philosophy-of-nature begins.[2]

[2] The Greek category of philosophy is quite different from ours, and they did not make distinctions between, say, an economist, geometer, physicist, or a political philosopher. All were about gaining true and certain knowledge of the things of the world. When we talk of

It was not long after the turn of the century that this delightful new 'naturalist' and rationalist (that is, open-to-reasoned-knowledge) project was subjected to the most excoriating criticisms by Parmenides, whose sceptical attack questioned the very possibility of obtaining sure and certain, timeless, necessarily true knowledge at all, let alone of the material or physical world. It may have been in response to this sceptical attack that the philosophers of nature noticed that amongst all the claims to sure and certain knowledge, the statements of geometry, or knowledge of geometry was in some way distinguished. Geometrical arguments seemed far more impervious to sceptical attack. The claims of geometry could be made sure and certain in a way different from all other kinds of claims, and could be rendered non-contingent and necessarily true by a form of supremely *reasoned* argument (like deduction), ultimately based upon the simple idea of constructing entities or validating procedures by circle and line construction. We call this proof, and it is something the Greeks invented … or discovered? … sometime in the first half of the 5th century BC. It is probably the single most important mathematical idea of all. The notion of *proof* by circle-line or ruler & compass construction means that geometry can be known about in a way that is different from knowing about physical things, and so makes geometry *epistemically differentiated*. Furthermore, starting from circles and lines alone and constructing a relationship using circles and lines alone makes the conclusions certain in a way that nothing else was at the time: in geometric reasoning the argument and the conclusions were made supremely clear and evident to the senses. Indeed, the concept of proof – a mechanism that gave a warrant for claiming geometry to be perfectly and absolutely true – made geometry the very paradigm of what we mean true and certain knowledge to be. In effect, in our understanding of nature, "true" means "true like geometry is true".

It is this paradigmatic certainty that so struck the early philosophers of nature, and it was the evidence of geometry – that there *could* indeed be necessarily true and certain knowledge – that gave the next couple of generations of Greek philosophers and philosophers of nature the grounds on which to build natural philosophical systems that could answer Parmenides' scepticism and could go on try to construct systems of knowledge of the natural world. Geometry, and the paradigmatic certainty of geometric proofs, is central to the early natural philosophical systems such as Democratean Atomism, Plato, and Aristotle, for example.

### *Plato*

Plato is the best known of Socrates' pupils (as Aristotle is the greatest of Plato's pupils), and whereas Socrates set up no formal institution for learning or for the promulgation of his studies, both Plato and Aristotle were to set up more-or-less formal 'schools' – the Academy and the Lyceum, respectively. Socrates' interest and his teaching were, to some extent, conditioned by Parmenidean

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Greek science and scientists, this is a misnomer and these are categories that were quite alien to Greek thinking. They were all philosophers, or perhaps philosophers of nature. This classical set of categories only broke down definitively in the 18th century.

scepticism, and were mostly directed towards an extraordinarily careful examination of the *conditions* of knowledge. He espoused no natural philosophical doctrines as such, but his teaching on the nature and conditions of knowledge are the foundations upon which Plato was to build – amongst other things – a philosophy of nature.

Plato argued that there were two kinds of reality[3], or two states of 'being', with (1) the world we live in, the physical, material, changing world constituting one kind or state or reality, and (2) a higher *other* kind of reality that was transcendental, non-material, changeless, perfect, and consisted only of ideal non-material Forms. These Forms are the perfect essences or models (like 'chair-ness' or 'blue-ness' or 'triangle-ness') that individual material things imperfectly manifest. Understanding of the physical world was to be had by understanding the perfect ideal Forms upon which the imperfect physical world was modelled; knowledge of individual physical things could at best be imperfect and changing (like the things themselves), but knowledge of the Forms could be perfect and unchanging (like them) … and therefore could be true, and timelessly true. Indeed, true in something like the sense that geometry was true. One of the ways Plato argued for the existence of this dual kind of reality was with the use of geometry: we all know that when we draw a triangle in the sand it isn't really a triangle because its edges are not really lines, the lines are not really straight and thin, and they certainly don't intersect at points. But the individual triangle we draw in the sand is taken to be representative – or a way of talking about, or thinking about – what triangles are really like. But … what is that? Of what is the triangle in the sand a rough individual copy? Plato suggests that the archetypical Form is something like the 'real' triangle or other mathematical entities: something we can think about and conceive of, if we work at it, and even reason about and gain knowledge of … but which is non-material and clearly not something that we can find 'existing' in the material world. It exists in a different, other kind of reality, and we use pure (logical, geometric) reasoning to gain sure and certain knowledge of it. So when we talk about triangles, and prove theorems about triangles, we're really talking about something like the Form of triangle.

For Plato geometry is a convenient and particularly *clear* way of illustrating what the dual nature of reality is like, and because the knowledge of geometry is so certain, a way of giving us a feel for the kind of that is certainty is available to us when we gain knowledge of the entities of his higher reality of Forms. What is *really* real – and what is the object of true, certain, and timeless knowledge – is a non-physical state of being, one for which geometry has privileged access. What we might call 'science' in Plato's world was to be something that looked and behaved like geometry, in that it achieved something like geometry's truth status. However, we should be careful: for Plato himself, the study of geometry was only a means to an end, it was not much of an end in

[3] It is, by the way, very easy to read Plato in a way that makes his whole model sound very fanciful and wild, but it is also easy to read it in a way that makes it sound deeply coherent with much of the larger metaphysical basis of modern science. Both are dangerous and historiographically dubious. The only sensible way to look at it is to try to understand it as reasonable within its own context, and not try to compare it to our own physical thinking.

itself. Plato is usually portrayed as being a champion of mathematical science, and certainly later readers of Plato were regularly (and fruitfully) to misinterpret Plato's natural philosophy as advocating the *mathematical* study of nature, as if nature were itself ultimately mathematical and mathematics was the way to reveal its true inner reality … and ultimately, the kind of mathematical physics the we have done for the last few centuries. In truth I think Plato cared rather little for geometry, as it was for him just the best way available to illustrate what he thought the experience of a true understanding of the really real objects of the physical world would be. Once one had grasped *how* to know this true reality, Plato suggested that one would carry on in the study of the Forms by other means only analogous to the method of geometry. I very much doubt the legend that he had "let no one ignorant of geometry enter here" inscribed over the door of the Academy; more likely it was something like "23b Acacia Avenue" (or, since this was downtown Athens, "No parking in front of these gates").

But geometry was a crucial step in Plato's analysis of our ways of gaining knowledge of the physical world, and this is most dramatically illustrated in the role geometry plays in his hypothetical school for Guardians. I say hypothetical because I do not believe that Plato thought for a moment that such an institution was a practical proposition, nor that its products would in practice make for the benign and benevolent absolute rulers that he hypothesised would be the ideal way to rule the state (rulers who could have morally justifiable absolute power because they possessed complete wisdom and knowledge of the essence of the Good, the Just, and the Beautiful). In the *Republic* Plato's project was an analysis and an argument, not a plan.

The similarity of the curriculum of the Platonic School for Guardians with a certain old-fashioned view of Cambridge education is, however, striking. At school one learns the normal, necessary social arts – reading, writing, arithmetic, fencing and the military arts, dance[4], history, literature, and so on. Then one turns up at the undergraduate stage of the School for Guardians – Cambridge – to study mathematics: arithmetic, geometry, astronomy, and harmonics,[5] so as to learn what pure and certain, abstract, non-conditional knowledge *is*. Not because the mathematics is useful, not because the Guardians (or the members of the British establishment) need to calculate, but because this trains the mind to know what knowledge, true knowledge, really is. Then upon graduation the tyro Guardians go on to do post graduate work, where they study the Forms that matter for a Guardian, but using a method of study analogous but even more certain than geometry: dialectic.[6] Using this higher form or reasoning eventually – for those who get Research Fellowships, perhaps, they come to know those things that a statesman needs to know:

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[4] This is a constant in gentlemanly education until the mid $18^{th}$ century, interestingly enough.
[5] Perhaps one should say "arithmetic and its application in harmonics, and 3-dimensional geometry and its application in astronomy."
[6] What exactly Plato meant by dialectic – or 'pure reason' – is not clear (because Plato did not know precisely what this was), but he probably had in mind something like what we call logic: the essence of the reasoning method used by geometry, but without the encumbrance of geoemtry's 'starting points and reference to circles and lines and other geometric entities.

about the Good, the Just, and the Beautiful. Once these post-graduate studies are finished, they can go out into the field as mini-despots in a sort of civil service (not accountancy or management consultancy – that is the mistake our modern graduates make!) and begin to work their way up the ladder of the ruling class.

In this vision of the nature of the physical world (and the good governance of the state) geometry played a crucial role, the role of a special and available window on the higher reality of the Forms, and as a training for the mind to know what was true knowledge. Once the study of geometry had taught one how to gain knowledge of the Forms and given one the experience of this supreme kind of true knowledge, Plato's own interest in geometry diminished. Crucially, however, Plato's natural philosophy was subsequently read as saying that true reality was in some sense mathematical, and that geometry was a special window on that perfect ideal reality. Thus the mathematical study of nature, or finding geometric models and relationships in nature was the distinguished way of uncovering the true an deepest nature (i.e. Form) of the phenomenon.

This is the next ingredient in the story of the role of geometry: once geometry became the paradigm of truth, we have the Platonic idea of geometry as a window on a higher reality. Moreover these ideas, despite a completely non-Christian origin and development, were later to be perfect for re-interpretation within a Christian cosmology that reinterpreted the Platonic ideal realm of Forms as the Christian God, or God's mind, or God's perfect realm. However, that is a later story, and I will return to this Platonised Christian view (or Christianised Platonic view, depending on which tail you think wagged which dog).

### *Aristotle*

Like all good pupils, Aristotle took on board Plato's ideas, developed them and toyed with them, and then rejected or profoundly modified the central ideas. Aristotle rejected Plato's dual ontology and rejected the ultra-mundane idealism inherent in Plato. Aristotle was, above all, an empiricist, arguing that it is only through direct study, observation, and analysis of the changing material world will we be able to understand the unchanging and timeless *laws* that its natural changes obey.

Aristotle had little role for mathematics in understanding the world; for Aristotle the way to understand the world was to know the underlying (abstract) universal principles or laws that governed the natural behaviour (or behaviour without external fetters or constraints) of natural phenomena. For Aristotle the distinction between natural action or properties and those not natural was crucial, and law-like behaviour – and therefore sure knowledge – was available only for natural phenomena. The renaissance catch-phrase was that "there is no science of the artificial". It seemed obvious to Aristotle that very few physical entities could hold mathematical properties *naturally*,[7] because almost all physical things have the property of change (they come into being and pass out of being), whilst mathematical entities like circles and lines have no inherent and natural property of change. A piece of paper may be flat but upon bending it, it is no longer flat but it *is* still the *same* piece of paper. So the paper held the property of flatness contingently, or accidentally, and not as part of its essence, or naturally. Generally, formal models and quantitative information seemed to him to have little bearing on the understanding of the natural qualities and causes of phenomena.

Rather, for Aristotle, the significance of mathematics – of geometry in particular – was in that special way of reasoning of the geometers that made their conclusions so apparently completely secure. Aristotle is yet another in a long, long line of thinkers (that goes all the way down to the likes of Marx and other recent thinkers and system builders, by the way) who like children in a playground have gone over to the playpen where the geometers were playing to find out what it is that makes them so cocksure of what they're doing, and then borrow whatever this is, and try to use it in their own games so as to achieve the same security and arrogant certainty as the geometers. What each philosopher or system builder sees as the origin of the geometer's security is different, but whatever they deem it to be, it is transferred to and transformed in their own natural philosophy.

In Aristotle's case, he felt that the key to the certainty of geometry was the ordered and hierarchical deductive systems based on distinguished starting points that the geometers around the Academy were developing. Not only did geometry reason logically or deductively, but it did so in an ordered, hierarchical manner, and from well defined, explicit and clear *starting points*.[8] He tried to adapt the deductive method of reasoning of the geometers to a deductive method that encompassed a larger (quantifier) logic, and the hierarchical deductive systems into a model for physical theories based on first principles and a strictly deductive schema.[9] Aristotle's ordered and deductive

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[7] … very few, but there were exceptions: Aristotle discussed the cases of three kinds of physical phenomena that could hold their mathematical properties essentially and naturally. These were the three 'scientia media' or mixed sciences of astronomy, optics and harmonics. However, these exceptions where we could gain true and certain knowledge of a physical phenomenon via mathematics *and* via empirical means could be explained reasonably well within the Aristotelian system, and remained tolerable exceptions to the rule. The key, in all three cases, was that the material *stuff* of the phenomena – the heavenly bodies, light, and whatever sound was (an activity of the air? A phenomenon of the sensate brain?) – was different enough from ordinary physical phenomena, so it could be argued could hold mathematical properties in a different, essential way.

[8] It should be noted that 'starting points' were just that: the starting points of chains of logical deductions. They were not (in the mid to latter 4th century BC) axioms or anything like a single small set of self-evident and non-redundant truths.

[9] I am being careful here: the geometers that Aristotle was learning from did not yet have what we call a (Euclidean) axiomatic system; these are the mid 4th century BC geometers who were probably the *sources* from whom Euclid was to develop his own complete axiomatisation of arithmetic and geometry sometime early in the next century. Aristotle's model of sure and certain deductive science as outlined in the *Posterior Analytics* was not a fully and properly axiomatic system either, although it goes some distance down that road.

system was not meant to be a method of scientific investigation or procedure, however: it was a model or a standard which a scientific investigation needed to achieve if it was to be a proper, sure-and-certain, necessarily true science.

In other words, Aristotle once again took geometry to be the model of sure knowledge, once again took the natural philosophical ambition to be 'true' to mean 'true-like-geometry-is-true'. For Aristotle geometry was not a distinguished way of uncovering the truths of the world, as it was for Plato, but geometry was the epistemic paradigm which a causal study must achieve in order to be a true and certain science.

### *Euclid and others*

Using Plato and Aristotle as two non-mathematical witnesses to 4$^{th}$ century BC geometrical thinking, we can answer an interesting question that is all too rarely asked about Euclid's *Elements*, probably written earlier in the 3$^{rd}$ century BC … and that is: *why*?? Why would anyone (in their right mind, anyway) have willingly written up a complete axiomatisation of geometry? This is not the sort of thing that people – even mathematicians – are led to do *naturally*. There had to be good (and even pressing) reasons to do something as painfully difficult and nit-picking as reconstruct the disparate bits of elementary geometry and number theory into a single geometrical axiom system. Of course, subsequently we think we know why it would be useful to have such a piece of mathematics (preferably done by someone else, as axiomatising complicated systems is brutally hard work, and rarely very mathematically interesting), but it is not entirely clear why anyone would have thought up such an idea in the first place.

So what question, what problem does the *Elements* answer? The answer, I would guess, is the way in which the truth and certainty of geometry, from the late 5$^{th}$ century BC, was becoming a more and more significant foundation for all natural philosophical thinking. It was being treated as paradigmatically certain. It was what the supremely true knowledge of the Forms was like, or it was the model of a true and deductive science. But was it really so certain? The assurance that what we have as geometry really, really is as sure and certain as we are claiming it to be becomes more and more important … important for the natural philosophers, as much as anyone else. The answer to the question of *why* anyone would write the *Elements* is probably that the need for an explicit demonstration of the paradigmatic certainty of geometry had become a natural philosophical imperative: if geometry was the paradigm of truth, it had to be demonstrably true, and so built upon a base that was demonstrably sure and certain. Axiomatics, the *next* great discovery in geometry, was the answer: demonstrably rigorous reasoning built up from self-evidently true and elementary starting points.

A couple of generations after the time when Euclid may have lived, about the middle of the 3$^{rd}$ century BC, another crucial idea was introduced about geometry, although an idea that seems to have been only very rarely appreciated by the ancients. This is in the work of Archimedes, who appears to have been quite autonomous of the concerns of the 4th century natural philosophers in Athens, and independent of the Euclidean axiomatic project as well. In one aspect of his work[10] he both contradicted Platonic Idealism *and* the natural / non-natural distinction that was at the basis of Aristotelian empirical science. In his applications of geometry to statics he demonstrated that we *can* have sure and certain, proved, geometrical knowledge of changing, material, artificial physical things. In these studies he gave a geometrical demonstration of the law of the lever or balance, for example. This is remarkable in the context of ancient natural philosophy: a geometrical demonstration of the action of an artificial mechanical device gives demonstrated, sure-and-certain knowledge of something that is far from natural in the Aristotelian sense. Archimedes was probably not the first to deal with simple mechanical relations using geometry, but proving the law of the lever or balance *out of geometry alone* was a radical step. The security of geometry was available in a new subject area. A geometrically demonstrated mechanics was a direct contradiction of Aristotelian natural philosophy and a strange anomaly for a Platonic natural philosophy. However, over the next 700 years of antiquity, only a few commentators seem to have noticed this new role for geometry.[11] It was deeply appreciated in the 16th century, however.

Plato's influence was the more pervasive natural philosophy over late antiquity in the Greek end of the Mediterranean, and it was Platonic thinking (often in its more mystical versions) that was to heavily influence the doctrines of the new Christian religion that proved so popular around the Mediterranean in the 2nd century AD and subsequently. The problem was simply that the early stories and texts of the Christian religion were far too intellectually, philosophically, thin to meet the kinds of standards of theological and philosophical thinking of Hellenistic (or Hellenised Roman) intellectuals. Thus as the paleo-Christian church developed, the participants in many of the early debates about doctrine – and they were endless and all pervasive – frequently turned to contemporary philosophy and natural philosophy for tools and resources. Particularly in the Greek east, these were predominantly Platonist philosophical tools.

The doctrines of the Church Fathers, the Patristic authors, are riddled with more or less explicit uses of Platonic or neo-Platonic arguments and ideas. And particularly important fort the early western (Pauline, Latin, Roman) Church was the use of Platonising arguments by St. Augustine, as his teachings were to form much of the intellectual basis of western theology through the early middle ages. Plato's doctrine of an ideal realm of Forms, after all, could easily be understood and adapted in Christian terms, and the creation 'myth' in

[10] In Archimedes' two principle papers on 'applied geometry': *On the equilibrium of planes* and *On floating bodies*.

[11] This is probably because Archimedes' work itself circulated rather little amongst late Hellenistic intellectuals except for the most mathematically literate. It was so technical that only rare mathematicians seem to have been up to the task of studying it, and these mathematicians seem generally not to have been alert or sensitive to the larger natural philosophical implications.

Plato's *Timaeus* is suspiciously easily Christianised. This had great influence in the 15th and 16th centuries, and I will return to it below.

Plato's doctrines also had influence on the late Roman philosopher Martianus Capella, who was one of the earlier philosophically minded educators who discussed the proper philosophical curriculum, and who was one of the originators of the hypothetical medieval curriculum that has come to be known as the Seven Liberal Arts, made up of the Quadrivium (arithmetic, geometry, harmonics, and astronomy), and the Trivium (dialectics or logic, grammar, and rhetoric). Although probably not practiced in the ancient world, this curriculum became a sort of ideal university curriculum in the later middle ages.

### *Arabs*

But before I come to geometry in the Latin middle ages, I must first loiter a moment with the medieval Muslim Arabs. The contribution to the development of Western European civilisation by the extraordinary flourishing of Muslim-Arabic civilisation from the 9th to 14th centuries is something systematically ignored or underestimated by our historical vision of ourselves. The Muslim Arabs did not merely transmit some classical Greek natural philosophical texts to the Latin west: they transmitted vastly more (advanced) technology to a technologically poor and un-ambitious society and they transmitted particular aspects of ancient Greek learning in their *own* interpretation and development.

The penetration of Greek technological, natural philosophical, and mathematical learning into the Roman Latin west had never been very extensive or deep. Following the half-millennium of civil (and intellectual) collapse that followed the decline of the western Roman Empire and the destruction of the *pax romana* in western Europe, what survived in the Latin west was often rudimentary indeed. After the Christian re-conquest of Moorish Spain an through Arab-western contacts in Sicily as well, from the middle of the 11th century Latin medieval students and scholars began their discovery of Arabic technology and applied arts, and then their discovery of some of the learning of ancient Greece and Rome. However, the medieval scholar had almost no intellectual tools with which to begin to read, translate, grasp, comment upon, and teach Euclid, Aristotle, Ptolemy or Galen, and even when the texts were translated into Latin, they had to read them through the eyes of Arab learning and the Arabic interpretation of ancient philosophy and natural philosophy.

For their own philosophical and theological reasons, the Arabs had found little sympathy with dualist, idealist Platonism and Platonic natural philosophy, but in Aristotle and Aristotelian natural philosophy they found the perfect companion to their own intellectual and theological interests, and so developed and taught what they inherited from the ancient Greeks in a deeply and profoundly Aristotelian way. This had pretty much infinite consequences for Western learning, not the least in the way that Aristotelian natural philosophy was to come to dominate western thinking from the 13th century. Particularly important here was the way that Aristotelian philosophy was to dominate Christian thinking after the 13th century Dominicans Albertus Magnus and Thomas Aquinas, overshadowing the earlier Hellenistic Platonic influences.

The second reason that the Arabic transmission and development (not corruption, *pace* the renaissance!) of classical Greek thinking is so important is one small, titanically important idea the Arabs added. Arabic Islamic thinkers, from their own theological arguments, had the idea, the extraordinary idea, that nature had been created by Allah *for us*, and that the world and His creation had all been put into place *for us*, for our dominion and exploitation. Thus the sciences in general, and most particularly the highest of the sciences, geometry, were given to us by God so as to be able to gain power and dominion over nature. Our learning about the nature and causes of the world was a divinely ordained and sanctioned enterprise to enable us to dominate, subdue, exploit and use nature. Put simply, this is the idea that natural philosophy (and particularly geometry) is utilitarian, and has practical and useful ends as its goal.

One can positively hear Plato and Aristotle turning in their graves as such an idea: for the ancient Greeks the idea of a utilitarian science was pretty much a contradiction in terms. This is *also* something that would have found little sympathy with Hellenistic Christian thinkers, who generally took corrupt, impure, and tempting nature as something best denied and best transcended in one's striving for the divine. The mortification of the flesh of the early Christian hermits is a good symbol for the early Christian view of the utility – and holiness – of knowledge of nature. Not so for the Arabs, and not entirely so for their distant pupils, medieval scholars.

## *The Later Medievals*

One has to have pity on medieval scholars of the 11th to 13th centuries – or more properly, we should be breathless in our admiration of their intellectual strength and courage. They were handed a mess: at the start of the 11th century rather little coherent learning survived in the Latin west from the ancient world, and they had few tools with which to deal with the material that they came into contact with from the Arabs. The intellectual patrimony of the half millennium between the end of the western empire and the beginning of the medieval contact with the Arabs was a small number of Christian texts without a great deal of intellectual apparatus to deal with them, and a small number of late Roman texts and encyclopaedias or compendia that covered only a tiny part of the textual wisdom of the ancient world. Then with the Christian re-sonquest or Moorish Spain medieval scholars come into possession of a load of (pagan) Arabic texts which included – sometimes interpreted, sometimes just translated – the wisdom of the ancient Greeks and Romans, all of it without a proper context and proper teaching programme. Once they started to make sense of the Arabic Greek texts, they were presented with a huge body of knowledge (especially Aristotelian natural philosophy) that was clearly a vastly better way of understanding nature than the rather pitiful early Christian natural philosophy that they had available to them … although this (apparently pagan) Aristotelian material was not always particularly coherent with Christian doctrines.

The way the medievals dealt with this was nothing short of brilliant, and it is their synthesis (or at least way of making sense of all of this) that is the foundation of subsequent western culture: it is the medievals who found a way of bringing together what have been called the three pillars of western culture: Athens, Rome, and Jerusalem.[12]

They dealt with the Greek scientific heritage with two brilliant ideas: what we call the doctrine of two kinds of knowledge, and the doctrine of the two holy pages. We would do well not to scoff at these doctrines, as in their evolved forms they remain embedded in our culture and thinking to this day, and motivated geometrical study for some 6 or 7 centuries, at least. I think many mathematicians today, in their pagan, agnostic, irreligious ways still echo these doctrines. The words have changed and the institutions have changed, but I suspect that the metaphysical intuition has not always changed all that much.

### *Two truths, two pages*

The doctrine of two kinds of knowledge or two truths was a simple slight of hand to deal with the possibility of obtaining knowledge of the world from a source other than holy Revelation. We obtain from Scripture or from Revelation (such as the Church Fathers and the writings of the Saints) direct knowledge from God, and that is the most true and certain knowledge possible. Our source of this knowledge, and our assurance of its supreme epistemic status, however, is based on our faith in God. Hence this kind of knowledge – and knowledge of the world – is *knowledge from faith*. However, God gave us the power of reasoning for some motive, and that is (in part) to investigate further the world and to obtain knowledge beyond that revealed to us by faith. This kind of knowledge is what the ancient Greek (pre-Christian, of course) natural philosophers obtained: it is knowledge obtained through logic and the exercise of our rational powers, so is *knowledge from reason*. Clearly – well, clearly to the medievals – knowledge from faith is superior and more certain than knowledge from reason, so should there ever be any conflict (for example where Aristotle's doctrines contradict the doctrines of the Church), then that from faith takes precedence over that from reasoning. Knowledge from faith comes directly from God, so is necessarily true, whilst knowledge from reason is mediated by the *human* process or reasoning, so is contingently true.

This dual state of knowledge fitted well with another early Christian doctrine that was to have huge and prolonged influence, and that is the doctrine of the two holy pages. This argument went simply that God has been revealed to us twice, and can be learned about from two different sources. First, there is the

[12] Just to be annoying, I would love to change this classic "three pillars" image and add the fourth pillar o f Mecca.

revelation of God in holy Scripture and other revealed writings (like that of the Saints). However, this is God speaking to us in the language of Man. Since we, our intelligence, our experience, our vocabulary and our thoughts are all infinitely less than that of God, inevitably the revelation of God in the language of Man is a cut-down, simplified, limited revelation.[13] Thus the revelation in the language of Man is clearly in code, allegorical and mystical, and needs to be unpacked and decoded (by the Church and the priest, not by you and me): Biblical stories are not to be taken literally, but must be decoded for the deeper teachings they contain that are inexpressible directly in the language of Man.

However, whatever God creates is imbued with God and God's purpose and meaning, so – this reasoning goes – clearly in Creation itself there will be God. Since we equate nature with Creation we can find God revealed in nature. The physical world contains – encoded in some way – a further revelation of God. However, this is a revelation that is not constrained by the limitations of the language of Man, so although the de-coding may be even more difficult than that of Holy Scripture, the ultimate message may be an even deeper and richer revelation of God. Thus is the study of nature sanctioned within Christian terms (something of which the paleo-Christian Church and the Hellenistic Church would never have dreamed), and nature seen as a source of knowledge of God.[14]

Clearly what we come to know of God through the revelation in the language of Man is knowledge from faith, whilst what is found revealed in nature was knowledge from reason. Thus did the medieval scholars establish a Christian rationale for natural philosophical study.

### *Studying nature*

However, the story continues, and is more subtle than this might appear. The arguments of the two truths and two holy pages give no method of study of nature, and no terms of analysis. Much of the energy of medieval scholarship was concentrated on this, and although the modern mind is unsympathetic to the whole enterprise, it was hardly a vacuous or unscientific enterprise … just not scientific in our own modern terms. Since the revelation of God presented to us in nature is encoded, clearly the first part of the enterprise of knowledge of the world, or knowledge obtained by reason, was to investigate the encoding of nature, or – as the medievals might have put it – to understand the theory of signs. The relationship between signs or symbols and the reality they signify became one of the core medieval interests, and it took them to levels of subtlety that a modern student can sometimes find extremely difficult to understand. And this is our key to understanding the importance of the Quadrivium and

[13] Imagine explaining the subtleties of quantum mechanics to a 6 year old child, only the case of God trying to show God's wisdom to Man is infinitely worse.
[14] From this it is clear why it was wise and sensible for the medieval, renaissance and baroque Church to keep an eye on the doctrines and claims of science, because the natural philosophers have a special handle on a form of Revelation.

Trivium for the medievals: for this is a programme in understanding *signs* and symbols.

The second part of the curriculum of the seven liberal arts is now more interesting to understand. The importance of logic to the medieval scholar is obvious: this is the fundamental tool to be used by reasoning to unpack our knowledge of the world, and logic (even if rhetorically written) is clearly a formal set of signs the manipulation of which gives rise to conclusions and knowledge that is potentially perfectly true. But grammar and rhetoric? Of course language is in itself one of the most obvious places where a theory of signs is used – we say 'cat', but we know it signifies the furry little beast sitting on our lap, and not a hamster or a dog, let along the colour green or the day after Tuesday. Language, like logic, is the quintessential place for signs to act. The medievals had the idea that there must be a deeper set of rules underneath grammar that was some sort of ultimate logic of signs and signifying, hence the enormous interest the medieval scholars had for centuries in grammatical theory. Similarly the rules of rhetoric (which at the time was more a set of informal rules in reasoning and the logical conveyance of ideas; the 'art of argument' might be a better way of describing it) was thought to ultimately reveal a deeper structure of ways in which signs carry and embody kinds of meaning that are not directly signified. So both grammar – as the formal set of rules of meaning – and rhetoric as the art of conveying meaning – were central aspects of the medieval study of the physical world, and thus the eventual understanding of the revelation of God in God's creation, or nature.

The significance of this should not be lost on us: whereas the early Christians had little place for the study of the world (it was either only the study of the corrupt material world and the realm of temptation, or it was irrelevant in the face of the direct knowledge of God and God's realm), and so little sympathy with Hellenistic natural philosophy or 'science', the medievals had good, profound, Christian reasons for doing something that we might call 'science'.

The first four of the liberal arts – arithmetic and its application in harmonics, geometry and its application in astronomy – were clearly not in there for the purposes of teaching more practical applied mathematics. Indeed, for all that the medievals studied geometry extensively and they had versions of much classical geometry from the Arabs, it is noticeable that they made rather little progress in what we would regard as geometric studies: there are few new theorems in Euclidean geometry uncovered in the middle ages. The reason is neither incapacity nor mathematical primitiveness. The medievals were perfectly happy to gut Euclid for the purposes of practical geometry – such craft practices as surveying or mechanics, for example, were quickly enriched with more geometrical calculating muscle-power as they learned more about geometry from the likes of Euclid and the Arabic textbooks in practical geometry. However, for the medieval scholar, something like Euclid's geometry was first of all useful for learning sufficient geometry to unpack the geometrical references in the Bible and the writings of the Holy Fathers, or in understanding the geometrical references in Aristotle. But for most medieval scholars, Euclid was a fabulous, wonderful and rich textbook in *logic*: it gave a complete working system of logic, and it gave a simpler (sentential logic) alternative to the syllogistic (or quantifier) logic of Aristotle.

It can be a little hard for us to appreciate this, but for the medieval scholar, the study of the pure, axiomatised, mathematical science of geometry was as much as a handbook of logic as anything else, as part of a vision of the world as working in terms of layers of signs and more-or-less hidden meanings. Like logic and grammatical theory, geometry was one of the paradigm examples of a world where mere appearances were not a true reality, but just tokens of a deeper divine reality.

### *New Texts: the Renaissance*

With the transmission from Byzantium of classical Greek texts between the 1390s and 1450s,[15] the bases of the medieval intellectual world were brought to an abrupt, almost violent end. The texts put in front of the 15th century scholar were reasonably faithful copies of the ancient texts the medievals had only had from the Arabs, texts that were still in the original Greek, and unfettered and uncorrupted by Arabic teachers and interpretation. Furthermore, what came over from Byzantium was not just the narrow range of texts that the medievals had had from the Arabs (mostly consisting of Greek medicine, astronomy, geometry, and Aristotle): there was a vastly greater range of texts. The influx of these texts was the ultimate cause of the renaissance, and in the case of the study of geometry, was to change everything, and change it utterly. In the first instance, these texts were much clearer and more complete than the Latin translations of Arabic translations of Greek mathematical texts,[16] and eventually – in the 16th century – decent Latin translations of the classical texts were to be made … and then *printed*, which mean that better and more reliable, fixed, texts were distributed far more widely. That, however, was a 16th century phenomenon. More important, however, were the changes that occurred earlier in the Renaissance in the very *status* of geometry.

One of the ongoing thorns in the side of the medieval scholars was a long series of disputes about the meaning or the nature of a number of Aristotelian doctrines that were simply obscure in the texts that they had available to them. As more and more texts in Greek or Latin translations from the Greek circulated (and were taught by Byzantine scholars deeply versed in classical philosophy), many of these disputes ended abruptly because the texts were clear or – as renaissance scholars learned quickly – one could understand a great deal more about Aristotle's doctrines if one read the later commentators, or his

[15] The transmission of texts began with a few texts that Petrarch had imported in the 1360s or so, but it became a substantive flow once the Florentine fashion for all things Greek began in the very last years of the 14th century. The flow dried up with the fall of Constantinople to the Muslim Turks in 1453, and the fall of Trebizond in 1456.

[16] Indeed, often the chain of translation of the Arabic texts was much longer than that, allowing for more and more layers of scribal error, errors of translation and interpretation and losses as a translator or copyist would ignore a chapter, or a proof, or a few lines, or mix commentator's comments with original text, or just not understand what he was reading, and so on and so forth.

teacher, Plato. So, whilst the medieval scholars had paid little attention to the few Platonic texts they had, earlier renaissance scholars quickly realised that Plato was a fundamental resource in understanding Aristotle. And no sooner had they begun to read (and translate) Plato seriously around the middle decades of the 15$^{th}$ century, than they realised that Plato was no mere instructor of Aristotle, but had the most powerful, elaborate and appealing set of doctrines himself. Moreover, there was an uncanny proto-Christian feel to Platonic doctrines … as if Plato had received a sort of pre-Christian revelation.

A pre-Christian revelation or prophesy is, of course, no problem: the Jewish tradition and the Old Testament are exactly that. So for some of a more mystical inclination, and there were many of that inclination in the renaissance, Plato gained extraordinary authority. He was an ancient Greek, and that gave his writings authority immediately. He was the teacher of Aristotle, which gave him a further vast authority. But he was also almost Christian in many of his ideas, as if he had had a partial revelation of God's laws and the nature of God's creation.[17] Of course what these scholars were hearing was only an echo of the early influence that Platonic thinking had on the Hellenistic paleo-Christian church and its early church fathers (most importantly Augustine of Hippo). These had been the intellectual roots of the Western church previous to the Aristotelian hegemony of the later middle ages. But Plato as a pre-Christian revelation, an authority in natural law like the authority of the Old Testament in moral law, made Plato in the eyes of some a natural philosophers quite unlike anything else. More than a century later, Kepler was to write that whenever he read Plato's *Timaeus*, he did so "on bended knee". This was holy revelation for the likes of Kepler. *That* is quite some authority.

It is not hard to find a renaissance Christian interpretation of most of Plato's fundamental doctrines. The dual reality is simply the difference between material creation and God's own realm, or God's mind, or paradise. In Plato's story of *the Cave* he tries to explain the epistemic difficulty of learning how to know about the Forms by illustrating it by imagining someone who has lived his life in a shadowy cave and then comes out into the full sunlight of a Greek summer's day, and is blinded by the Sun's light, and will only slowly get used to seeing in such bright light. This is easily interpreted as an analogy to the idea that we know all things only in the light of God's revelation, and that the Sun (giver of light and heat, and therefore life) is a symbol of the presence of God in the world.[18]

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[17] Just as others believed that Hermes Trismagestus, the supposed author of the Hermetic texts, had received the keys or commandments of *natural* law at the same time that Moses had received the commandments of *moral* law. It only emerged later that the Hermetic texts were written by a group of Egyptian Christian heretics in the 3$^{rd}$ or 4$^{th}$ century.

[18] You might note that this very 15$^{th}$ century idea very easily equates God the ruler of the universe with the Sun as the ruler of the world, and thus easily places the Sun in the more noble position of the centre of the world (and not the rather undistinguished classical position of 4$^{th}$ object away from the Earth, between Venus and Mars) … and thus implies a heliocentric model of the universe! This sort of conclusion had already been discussed by Nicolas of Cusa in the 1440s, in texts well circulated and read throughout neo-Platonist, humanist Europe. Copernicus' idea of heliocentricity, arrived at in the last years of the 15$^{th}$ century and published in 1543, was hardly making the radical innovation with which he is sometimes attributed.

And the special role of mathematics for Plato? This was easy to understand. By definition, God's thought, God's very thinking, is true. It could not be otherwise, by definition. And what is the one kind of reasoning, the one way of 'talking' that we have that is necessarily, absolutely, and paradigmatically true? Geometry and arithmetic. Of course God's thought goes vastly, infinitely, beyond geometry, but at least in geometry we had a special point of contact with God's mind.

Thus for some in the Renaissance (but by no means all), geometry became a special window on God's mind and God's thought. Where one could find some geometric model or geometric analogy in a phenomenon or a relationship, one was uncovering something of the deeper, hidden pattern of God's plan and God's thinking of the world. For some geometrically minded thinkers, this could lead them to deep geometrical investigations into the nature of phenomena that in today's anachronistic terms might look like mathematical physics, and for others, this lead to geometrical mysticism that looks to the modern reader to be as far from anything 'scientific' as could be. These, however, are the false readings of a distant scientific world: to contemporaries in the 15th and 16th centuries, they were merely part of a spectrum wherein God's very thought was seen to be revealed by the geometrical patterns and structures we might find in the world. Geometry became, for some, a holy mathematics, a divine science the study of which was a study into the very nature of God and God's revelation. This, of course, is just a continuation of the Two Holy Pages doctrine, in its renaissance, Christian, Platonic form. And it gave geometry and geometrical studies or arguments an authority and a weight they had never had before. The Christian misreading of Plato made geometry the ultimate science for many in the renaissance.

### *Archimedean Engineering*

However, there is another aspect of geometry in the renaissance that must not be overlooked. The texts that came from the Byzantines included not only the few mathematical and natural philosophical texts that the medievals had had from the Arabs, but vastly more – covering history, literature, politics, letters, geography, and the rest of the gamut of classical Greek literary production. Included in this were the a number of what, for lack of a better term, I might call engineering texts. Treatises by Archimedes, Pappus, Hero of Alexandria, Ctesibius, Vitruvius, and others gave more or less mathematical treatments of simple engineering problems, statics, the five simple machines[19], hydrostatics, and so on. Where the medievals had had versions of these texts, they had paid scant attention to them: this sort of applied mathematics hardly fitted in with academic geometry, and for practical purposes there were better Arabic or home grown treatises available.

[19] These are: the wedge or inclined plane, pulley or windlass, gear or axle, lever or balance, and screw.

Up to now, 'mechanics' had been a craft practice, a practical art that was the domain of the unlettered and un-gentlemanly. However, such was the authority of ancient Greek texts that as these 'engineering' texts came to be appreciated, the idea of a gentlemanly *scientia* of geometrical engineering emerged. What made it the proper domain and practice of a scholar and a learned, lettered gentleman was not only that it was Greek (and anything from ancient Greece was practically revered), but that it was based upon geometry, rather than laboratory or workshop practice.[20] In this, the treatises of Archimedes on mechanical and fluid statics became a sort of new paradigm: this was a geometric method to gain true and certain knowledge of the physical world. If one could prove the law of the lever or balance from geometrical considerations alone, then one could have a true mathematical *scientia* of practical things like machines.

The effect of this was two directional. There were the learned and gentlemanly who claimed that such apparently practical arts were – if done in the manner of the Greek geometers – the proper domain of the scholar and natural philosopher, and (more important, I think) there were practitioners who seized upon the idea of doing practical 'engineering' using the language and trappings of classical geometry, and this rendered their practical art or *techné* a gentlemanly *scientia*. It raised the social and intellectual status of their work, and it raised their own social status … at least, this is what they hoped! Geometry, where it was done in some sort of echo of the ways of the ancients, had the power to render a craft practice a natural philosophical study, and render its results not the conditional rough truths of the practical art, but the sure and certain truths of *scientia*.

To have true and certain knowledge of changing artificial and unnatural things like machines was, of course, a contradiction of Aristotelian natural philosophy. In the 16th century a number of natural philosophers considered this question, and concluded that the reason why we could have this apparent contradiction with Aristotelian ideas was that geometry was, in a sense, the ultimate empirical science – in that of all experiences, those of geometry, geometrical quantity, and geometrical reasoning, present themselves most perfectly clearly to our senses, and thus are potentially the most clearly, completely, and perfectly knowable. Therefore we are able to have perfect and completely certain truths of those things which can be expressed geometrically, whether they are natural or artificial. It was a good bandage, and in a world where your colleague down the hall at the university was probably a raving carpet-chewing Platonist, finding geometry and therefore God in just about anything and everything around you, it was a reassuringly sober way of explaining a reassuringly sober science like geometrical mechanics or geometrical engineering. It was not, however, a bandage that was to work for

[20] Of course a gentleman of the time would not dream of actually rolling up his sleeves and practicing mechanics, making machines or anything of the sort: the role of this learning was either only the show of wide and classical learning (an end in itself in 16th century courts), or as a landowner or gentleman of arms, in overseeing the mechanical practice and works of mechanical operatives. To be able to establish proper modes of separation of labour was crucial in the development of the new sciences in the 16th and 17th centuries.

long. For those who happen to know the life and work of Galileo before he became celebrated for the telescope and his Copernican views, you will recognise him as one of the many 'Archimedean engineers' or practical, Archimedean mathematicians working at northern Italian universities late in the 16th century. And for those who happen to know the early work of Mersenne, Descartes' mentor and friend through much of the 1620s, you will notice where Mersenne's faith in a mathematical mechanics came from.

## *Jesuits*

Around the same time, late in the 16th century, another interesting and revealing justification of the study of geometry is to be found amongst some of the toughest and sharpest intellectuals in all of Europe: the Jesuits. Jesuit education was (and continues to be) regularly slandered by Enlightenment *philosophes* and their followers as ultra-mundane, backward, unscientific, pedantically Aristotelian and Thomist, and un-modern. In general, however, the exact opposite was the truth from the beginnings of the educational story of the Order: where they could, the Jesuits gave tough, high-level, rigorous and up-to-date natural philosophical and mathematical education. They were educating what they intended to become an elite ruling class, and they knew perfectly well that being behind the times scientifically was not the way to achieve that. In their theology they may have taught Thomism, but in their mathematics and natural philosophy, they were pretty rigorously up to date.

Their justification for teaching geometry – which they did everywhere in the world, and often to astonishingly high standards – is interesting: abstract, rigorous geometry was to be taught because this was the very best way to train and exercise the mind to have faith and to have belief in the most extraordinary things, to be able to exercise a completely convinced faith in things that the mind cannot experience and cannot know by mere perception. In other words, once you had mastered the rigours of geometry, you had the tools to have faith in the deepest of Christian mysteries. This justification was used late in the 16th century, and we find it present in Jesuit arguments through the 17th century; by the 18th century, the argument became even stronger, as the 'geometry of the infinite' (be it infinite series or the infinitesimal calculus) became taught: teaching the mysteries of the geometry of the infinite was an excellent training for the mind to deal with the infinite mysteries of God and revelation.

## *The Scientific Revolution*

From the late 16th century to the early 18th century the changes in mathematics and in natural philosophy were so deep as to merit the dramatic label of 'revolution' … although there is no moment of storming the Winter Palace, no single set of doctrines that actors at the time could identify as 'revolutionary', no leader haranguing the throng. The 'revolution' is a construct of their successors, who were aware they had lived after an extraordinary convulsion in thinking about mathematics and science. Central, however, to whatever this 'revolution' might have been about was a change in the status of geometry – and indeed, a very change in the nature of geometry, as it became algebraic-geometry, and then became the 'infinitesimal algebraic-geometry of curves' or, more familiarly, the calculus.

However, the revolution was not always so revolutionary: the change in the status of geometry was the extension, the rendering almost universal of the ambitions of the Archimedean style of practical mechanical mathematics of the late 16$^{th}$ century. Two of its most publicly known practitioners were Galileo and Descartes, and their connexions with the 16$^{th}$ century Italian tradition are clear and simple.

## *Kepler*

However, we should not forget one of the last radical neo-Platonic mathematicians, at least 'last' before the Platonic idealist view of geometry was to mutate in the 17$^{th}$ century into a less mystical and less theological set of intuitions. This is Kepler, who thought that each moment he was doing geometry was a moment of direct contact with God's mind, and that in finding the geometric models of the motions of the planets, he was uncovering the exact language of God's thought. And this is Kepler who ascribed to Plato the decree that "God is a geometer".

In the *Mysterium Cosmographicum* of 1597 Kepler asked an interesting question we no longer think of asking – why there are the number of planets there are, and why their orbits are in the relative proportions that they are? The answer lay in the geometry of the Platonic solids: it proved possible to circumscribe and inscribe the Copernican spheres of the orbits of the six planets[21] in a unique way between the spaces occupied by the 5 Platonic polyhedra. Barring a small but nagging problem with the orbit of Mars, this perfect modelling of the unique Platonic solids and the orbits of the then known planets around the Sun was, for Kepler, both a justification for the Copernican model and an explanation of the order and number of planets. God had simply had the Platonic solids in mind when the orbits of the planets had been ordained, so five solids meant there could not be seven planets (as in the classical geocentric model), but only six.

The niggling error of Mars that seemed to have too thick a sphere, leaving too much useless space between its circumscribed and inscribed solids left Kepler dissatisfied. God, after all, didn't make errors or clumsy bits of physical geometry and would not leave needlessly empty space: anything made by God was both perfect and purposeful. Hence, eventually, what Kepler was to call his "war on Mars" to better understand its orbit, and then by a series of fortuitously compensating errors, he uncovered the elliptical orbit of Mars and thus the rest of the planets. He was able to find some tentative (physical) justifications for God choosing elliptical orbits in his *Astronomia Nova* of

[21] In the classical ancient models, such as the Aristotelian and Ptolemaic models of the heavens, there are seven planets revolving around the Sun; in the Copernican model there are only six - Mercury, Venus, Earth, Mars, Jupiter, and Saturn; since the moon is no longer a planet of the Sun.

1609, but it was only in his *Harmonices Mundi* of 1619 that he finally found the true reason for God's creation of the universe in the form that we find it. Kepler was able to show that the ratios of the maximum and minimum velocities of the planets, intrinsic properties of the orbits, were in ratios and proportions that were harmonic, musical ratios. Of course it is only with the ears of the soul (or the mind of the geometer) that such celestial harmonies can be perceived, and the music of the heavens thus enables man to find (or hear) God through God's perfect, geometrical creation.

For Kepler, the *physical* cause of the elliptical orbits was two fold: firstly an immediate material cause (which he speculated was an active power of matter something like magnetism), but more important was the divine cause, or the motivation which persuaded God to make the universe in a particular manner, which we could understand at least in part through geometry. The mathematical harmonies of the geometry of the heavens was in itself a kind of higher, theo-physical causation. For Kepler, the mathematics of God was the true physics.

Kepler's vision of geometry as a method of perception of God and as a connexion of Man with God was an intuition that had had a long appeal to the renaissance mind. However, such a mystical view of the workings of the physical world, and such a theological view of geometry was soon to fall out of fashion in the 17$^{th}$ century, replaced slowly by a rather different vision of the relationship between geometry and the physical world.

### *Galileo*

Both Kepler and Galileo wrote something very near to the words "the book of nature is written in the language of mathematics". Both would have described themselves – or the mathematics they used – as geometers, and both meant that geometry was the key to understanding nature. But the irony is that they meant totally different things by this phrase. For Kepler, the book of nature was written in mathematics because mathematics was a unique window we have on God's thoughts. For Galileo, the book was written in mathematics because mathematics – geometry – perfectly described or characterised nature: nature acted or expressed itself in a way that geometry could capture.

If one important strand of renaissance thinking about geometry was the neo-Platonic vision of God as a geometer, another was that of those renaissance Archimedean practical mathematicians with epistemic ambition. This was a view that saw geometry as a way of modelling phenomena, and indeed could *perfectly* model them. Geometric calculations on such models had physical significance; the phenomena were not themselves inherently mathematical in a Platonic sense but the laws that governed their behaviour were in some way expressible using geometry.

The elements of a geometrical mechanics had been slowly sorted out over the 16$^{th}$ century, and Guidobaldo del'Monte's *Liber Mechanicorum* of 1577 had pretty much completed a geometrical description of simple statics and the compounding of the 5 simple machines. Guidobaldo, by no coincidence, was one of Galileo's early patrons, and someone to whom Galileo paid the greatest respect (which was a rarity for Galileo). The next problem – and I think it was clear to late 16th century students of this sort of Archimedean mechanical study – was the problem of motion, of projectile motion. This is the sort of forced, artificial motion that Aristotle's physics had left *outside* rigorous and certain science, but by the last half of the 16th century it seemed commonly agreed amongst a small sector of artillerers, applied mathematicians and academics that there *ought to be* laws of projectile motion. This was the central theme of Galileo's early scientific work, and it was only by accident that he was ever distracted into the Copernican debate.[22]

For Galileo (and others of this Archimedean style), there was no higher ideal reality, nor were they talking to God when they did geometry. Instead geometry was a tool: a special tool, and a tool that allowed true and certain calculations, but still only a tool. Nature and material phenomena were messy and imperfect in appearance because any phenomenon was a complicated combination of a large number of different component effects – and geometry, geometric analysis and geometric models allowed for the separation and unpacking of this complicated compounding into its unfettered 'perfect' component parts. Once the 'perfect' and pure parts had been separated and isolated by geometrical analysis, they could be studied and modelled by geometry because their behaviour – now perfect – would be such things as motions in perfect straight lines, or curves, or whatever. The imperfections of appearances was only the result of the messy super-positioning of many phenomena in one single physical activity.

This is the triumph of Galileo's analysis of free fall and projectile motion: after lengthy investigations, he found the extraordinary *geometric* answer to the nature of projectile motion. It was made up of simple, constant, and easily decipherable and understood components: a constant horizontal motion and a constant vertical acceleration. If the appropriate geometry is used to break the complex into its components parts, we can isolate the 'pure' phenomena of nature using geometry, and then study their 'pure' behaviour in apparent isolation, using geometric idealisation now with validity, as we are dealing with the pure phenomena rather than compounded phenomena. A linear motion will not be nearly linear – we can conceive of it as perfectly linear plus some other effects (bumps on the table, table imperfectly flat, or whatever). Each of

[22] Galileo's work in Pisa and Padua (from the late 1580s to 1610) was generally around what we would today call applied mathematics, Archimedean statics and some kinematics, and engineering. Galileo augmented his meagre salary as a professor of mathematics by coaching students ("who make my head hot", he wrote … showing how *some* things never change), undertaking some engineering consultancy work – as we would say today – for example on the draining of the Paduan marshes, and making and selling various mechanical devices. These latter included an artillerer's compass (for calibrating and ranging cannon) and, fatefully, the spyglass, which was conceived as a military tool until Galileo turned it to the heavens and made observations of a natural philosophical nature with it. Only when the spyglass – telescope showed him clear evidence in support of a Copernican cosmology did Galileo's ambitions grow from a frustrated Archimedean applied mathematician to a mathematical natural philosopher.

the component parts can now be studied as an ideal limit, and so was open to the use of a perfect geometrical language.

So Galileo uses geometry to break down the appearances of physical phenomena into their component parts, and then is able to use geometric and quantitative analysis to provide geometrical laws of behaviour. Geometry is the perfect tool for the study of nature, but there is no claim that the material phenomena of nature are themselves in some extra-physical way 'geometrical'. Using this type of Archimedean geometrical analysis Galileo was able to establish a new science of projectile motion, and this, in itself, is the most profound contradiction of Aristotelian natural philosophy. It was this 'new science of motion' (his geometrical analysis of projectile and circular or orbital motion) that gave him both arguments to answer the objections to a heliocentric, geo-mobile cosmology *and* gave some supportive arguments as to why the universe was heliocentric, and we don't sense that we are rotating and orbiting at such great speeds.[23]

It should also be noted that Galileo had another, polemical use for geometry in his science: when he was arguing in print with the Aristotelian natural philosophical position he was at pains not to begin from argumentative positions that the Aristotelian would not accept. Hence, for example, Galileo does not claim to have found out various things from experiment because he knows the Aristotelian could always argue that such experiments were artificial physical situations, and so would not reveal nature acting naturally – and so the results of such controlled experimentation could always be denied as not illustrating truly natural laws.[24] When faced with such obstacles, one of Galileo's polemical tactics was to set up the physical argument in geometrical form, and take his Aristotelian interlocutor through an argument that was wholly geometric in its appearance. Thus when he arrived at a conclusion that the Aristotelian didn't like the Aristotelian was apparently logically obliged to accept the conclusion, because geometrical arguments were unassailable, a perfect form of logic.

### *Descartes*

If Kepler illustrated empiricist the inheritance of that mystical dualist Platonist view of geometry, and Galileo the inheritance of an Archimedean mechanics by geometry, then Descartes – the third key figure in the scientific revolution – showed something of an inheritance from that Archimedean mechanics as well as the Aristotelian answer to it.

[23] This may sound all very fine and good, but we should remember that the physical hypotheses of Galileo were not altogether successful: his analysis of free fall led him to conclude inertial motion was *circular* motion (not linear!), and he continued to use circular orbits for the planets not ellipses (more than twenty years after Kepler's publication on the elliptical orbits) because his kinematic and dynamic intuitions simply could not stretch to the ellipse. Geometry may be a good tool for analysing physical phenomena, but it doesn't make for good physics *a priori*.
[24] This is the origin of his famous strategy of "thought experiments".

Descartes had the simplest of ideas: in re-thinking how to understand physical phenomena, he postulated that matter came in ultimate small corpuscles of various sizes that filled all space, and that all material phenomena were caused by nothing more than the compounded and complex motions of corpuscles. These corpuscles had no properties whatsoever except that they occupied space, and then when hit by something, moved off according to simple billiard-ball mechanics until it was hit by something else. With such an austere ontology, Descartes was able to derive *linear* inertia[25] in about three lines of reasoning.

Since all we can know about these corpuscles are their position and motions, which are only geometrical quantities and can only be calculated with in a geometrical manner, Descartes decrees a material universe which is ultimately only geometrical. That is, all secondary qualities and appearances are only derived from some ultimate or primary phenomena which are completely knowable in geometric terms. This was the most radical reductionist natural philosophy since ancient Atomism, with appearances so separated from their ultimate causes. The mechanics of these particles is a billiard-ball mechanics which Descartes took to be an extension of Archimedean static mechanics.[26] So, Descartes claimed, all physical phenomena can ultimately be reduced to a geometry of particles banging into each other. The clever part of this programme is that this ultimate particulate geometry was to be discovered using – amongst other methods – experimentation, so for the first time a natural philosophy placed the experimentation at the centre of its methodology. And one would know when experimentation – or other analytical procedures – arrived at sure and certain geometrical laws or first principles because nothing can be so clear to the senses as geometry, so the geometrical first principles of phenomena will appear to us as the most perfectly clear things that our reasoning or our experimentation can reveal. Geometry as the ultimate, perfect, empirical experience!

Descartes took a dualist ontological position: there are (1) non-material phenomena (like emotions or thoughts or other phenomena of the spirit) which have no material qualities or effects whatsoever (an idea his no weight or volume, and cannot move a pencil); and (2) material phenomena which are ultimately derived from nothing more than the motions of ultimate, inert material particles, and which have no powers, volitions, goals, desires, or other 'occult' powers: they only obey the laws of a geometrical mechanics. The power of Cartesian natural philosophy was that since all physical phenomena could be completely understood with geometry, and geometry was completely open to our understanding and supremely clear to our rational senses, all phenomena were open to reasoned and rational understanding, and that complete (geometrical) certainty about the *whole* physical world was therefore available to us. There are no mysteries, no occult and secret powers running the world, no teleology, and there is nothing material that necessarily escapes the geometrical grasp of human understanding. To a philosophical world

[25] That bodies are indifferent to their straight line motions, and will continue to move in straight lines until a collision changes their motion to another straight line motion.
[26] However, Descartes then gave a completely erroneous set of laws of billiard-ball particle mechanics.

deeply disturbed by the power of mystical and occult sciences, and the claims to true (but secret) knowledge by alchemists, Paracelsians, cabbalists, naturalists, Rosicrucians, hermeticists and so many others, in the 1630s this must have seemed a wonderful and safe natural philosophical refuge.

Phenomena of the spirit – from thoughts and emotions to God and the soul – were carefully left out of this programme as Descartes knew well enough not to take on the Church and not to lay claim to be able to arrive directly at an understanding of God or God's will. That was the domain of the Church, but Descartes did seem to have to posit a world in which once God had set the whole particulate system working, it appeared to be able to carry on working without any further intervention from God.[27] Furthermore, it was far from clear how the non-material soul could work on the physical body. In a Cartesian materialist world, the problem of determinism is suddenly very real, and that bane of the philosophers, the mind-body problem, is on the table. The nature of the transformation of the bread and wine in the Eucharist posed no small problem for a 17th century corpuscularian materialist. The accusation of practical atheism was not long in coming, and (Cartesian) materialism and atheism soon became conflated. Suddenly, geometry *wasn't* about God!

In some senses, we continue to live in a world where we look at the physical word in Cartesian mechanical, causal terms, although Cartesian natural philosophy and the role of geometry in understanding the physical world have changed deeply since Descartes. Determinism, free will, and the mind-body problem are all still with us, and still problematic. But the corpuscular, geometrical, mechanical view of Descartes soon failed because it turns out that attributing to matter *only* the geometrical powers of occupying space is insufficient to get the universe to work. Matter needs a few more powers (nowadays we call them 'forces' and try not to let them look too much like 16th century mysterious occult powers) to give rise to the observed universe. In a sense, this problem of the excessive paucity of the Cartesian ontology is the problem that Newton started to tackle.

### *Geometry as science*

However, for all that geometry played deeply disparate roles in the new physics of Kepler, Galileo and Descartes (and other figures of the scientific revolution), one thing remained constant: that the paradigmatic method of understanding nature was not just quantitative, but was through the use of geometry. Everyone agreed that in some sense nature *was* geometrical, or could best be understood or modelled geometrically, and that in uncovering the geometrical truths of nature we uncover something that is paradigmatically true and certain of nature. The certainty of the new mechanical science that emerged from the

[27] Descartes rectified this by noting that nature is under-determined by mechanics (that is, many – infinitely many – different mechanical processes can be hypothesised that give rise to the same physical phenomenon. Thus we need to do an *experimentum crucis*, a decisive experiment (not an analytical experiment) that tells us which of these possible mechanical systems God actually chose to make the appearances we observe. However, this fudge seem not to have had much of a mollifying effect on Descartes' detractors.

middle of the 17th century rested upon the certainty of geometry: nature ultimately behaves in a geometrically describable way, so just like mechanics, and then projectile motion, the laws of nature can be demonstrated geometrically.

This is a fantastically powerful claim: that the laws of nature are in some sense geometrical, and can be demonstrated by geometry. Of course we have all heard of how the new science of the scientific revolution finally put the experimental method at the heart of science, and it is indeed true that natural philosophers began to talk about experiments, and even sometimes did them. But this was generally an investigative tool, a way of uncovering the regularities of nature. Experimentation did not in itself carry the greatest epistemic weight, no matter how many of the great-&-good witnessed an experiment. Ultimate certainty came from the geometric proofs of the laws of nature. Mechanics was exhibited as the paradigm science, but it had this status because it was paradigmatically *geometrical*. Natural philosophy had moved from Platonic idealism or Aristotle's qualitative empiricism and teleology, moved from geometry's pattern of logic as an ideal goal of natural philosophical knowledge. The security and certainty, the ultimate truth of natural philosophy was achieved because it gave laws of nature that were geometrically expressed and geometrically demonstrated.

This is dramatically illustrated by Newton, for example in the final section of the *Principia*, where he is trying to sum up his grand model that synthesises a mathematical (terrestrial) mechanics based on the notion of force and his celestial mechanics, based on a completely obscure force called universal gravity. (Obscure because how such a force can operate over empty space was deeply unclear.) This is where Newton claims he will eschew hypotheses (the infamous *hypotheses non fingo*) as to the (corpuscular, mechanical) cause of gravity. Of course he wants to avoid hypothesising a cause because he still cannot give a mechanical cause of gravity – not for want of trying, mind you – and anyway, in the subsequent paragraph he will launch into some speculations as to the cause of gravity, so he keeps to his high-minded *hypotheses non fingo* for all of about 15 lines of print. However, he claims he has no obligation to offer hypotheses for the cause of gravity anyway, because he has been able to *demonstrate* the laws of gravity from the nature of Euclidean space and by geometry alone, so a causal mechanism will add nothing to the truth of his laws of gravity.[28] If at one level such methodological arrogance is untenable, Newton is also correct about his geometric demonstration of the basis of his celestial dynamics: if space is flat and Euclidean, then it is not hard to demonstrate that bodies act as point masses, and it is not hard to demonstrate

[28] *That* is whistling in the dark, and Newton knew it. Even if the laws of gravity *are* geometrically demonstrated, that doesn't deny the legitimacy of the question "by what mechanical means are these effects produced?" This is not a mere *metaphysical* question, as Newton sneers: it is a reasonable mechanical question. Such causal agnosticism, however, is a useful position to take and appealing when one does not have the faintest idea of what is going on causally, but has the luxury of a demonstration of the result by other and unimpeachable, geometrical, means.

that from a point mass a force must 'radiate' its power isotropically, and so with a $r^{-2}$ law.

So – in effect – celestial dynamics can be derived from geometry alone. *That* is a powerful argument!

### *The Enlightenment*

In a sense this is a fulfilment of the ancient Archimedean dream (although I wonder if Archimedes ever had such a dream in such terms) of using geometrically demonstrated laws of mechanics to model the physical world. It is also the nominal beginnings of a kind of physical-mathematical thinking that was to become the paradigm of the new physics of the Eighteenth century, called at the time Rational Mechanics. This was the simple idea that like Newton's laws of gravity, ultimately the laws of physics should be derivable from geometry and proved by geometry. One did not need to do experiments to *justify* such a physics (although experimentation might be the method for obtaining them in the first place), and one did not need hypotheses as to the nature of matter, the causes of forces, or other ontological hypotheses (hypotheses which were troubling and unclear to contemporary 18th century natural philosophers): ultimately the laws of physics could be demonstrated geometrically, without ontological commitment.

It was a nice dream, but it did not work: in the end, physical hypotheses are needed (for example, Newton took space to be flat Euclidean space, which turned out to by a physical claim, unbeknownst to anyone before Einstein). It was called Rational Mechanics in the 18th century, but it is only what the 19th and 20th centuries developed in to theoretical physics: a complex dialogue between ultimate physical hypotheses about the nature of space, force, and so on, and the mathematical analysis used to develop those hypotheses. The weight of demonstration, the epistemic warrant of certainty, however, still comes from the mathematics.

Descartes' dream of a new universal science based on an ontology that was ultimately only knowable through geometry turned out to be insufficient, because matter needed 'powers' to operate. The project, however, was not completely abandoned. Newton gave mathematical form to the laws that the force of gravity operates with, and so as other forces like electricity and magnetism were studied in the earlier 18th century there was at least some hope that whatever they were, they were not the old fashioned occult powers or teleological natural tendencies, but mathematically describable. There was a ghastly five or six decades from the end of the 17th century when natural philosophers had to deal with an increasing plethora of forces (electricity, magnetism, heat, light, pressure, friction, stress and strain, chemical powers, as well as the 'animate' powers of the parts of living things), and they had little sense of what these 'forces' were or how they operated. Then, from the middle decades of the 18th century some these forces started to fall to some sort of mathematical, geometrical, analysis. Thus even if the ontology or mechanical cause of these forces remained obscure, at least they behaved in a geometrical, law-like manner, and moreover, they behaved according to similar and analogous rules. One 18th century natural philosopher wrote that forces were incomprehensible but "time has domesticated them." Eventually, it was geometrical analysis that really domesticated them. Forces like gravity, electricity, magnetism, and so forth operated in a law-like way in space, they behave geometrically, and they obeyed geometrical laws. These forces were not mysterious because whatever they were, they were geometrical.

### *A more public face*

One of the dramatic consequences of the scientific revolution, and the whole change in the social status of the mathematical and applied sciences from the later renaissance on through the 17th century was the change in the public profile of natural philosophy. The age of the technical expert and technocrat, the scientist-as-sage had begun. Natural philosophy and its applications came out of the ivory towers and noblemen's courts, and started to become part of the economic and therefore the social fabric of lettered society. Voltaire wrote of his admiration of English society when, at the funeral of Isaac Newton he saw a "philosopher buried as if he were a King".[29] This was a new status for a mathematical natural philosopher; a century previously nowhere in Europe could a mere geometer and natural philosopher have been so lionised.

Geometry and the teaching of geometry were not unaffected by this new social weight and social role. Geometry was still the finest way to train the mind, the most perfect training in reasoning and clear thinking. It was still taught by Jesuits and others as the ultimate fitness training for our rational powers of faith and knowledge of the unknowable. But over the 17th century geometry gained a new stature as it became the underpinning argument of the new mechanical philosophy. Geometry was the symbol of ordered and reasoned knowledge, and a training in geometry was the surest buttress against any sliding backwards into superstition, ill founded beliefs about the world, and the anti-rationalist occult natural philosophical heresies of the late renaissance. Teaching geometry meant bringing up men[30] with the power of reason, and to reason about all things. To the (slightly) democratically minded thinkers of the Enlightenment the diffusion of a 'public' and popular power to reason became a foundation stone for a freer, more liberal and socially responsible society.

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[29] Mind you, Jonathan Swift witnessed the same event, and sneered that he despaired of such a fuss being made of 'a clockmaker'.

[30] There is, in fact, the beginnings of a rumbling argument about whether it was right or appropriate to teach geometry, or mathematics beyond household arithmetic, to women. There were many who held that the rigours of such abstract reasoning were alien, even hostile, to the gentle and caring femininity of "the fairer sex", but there were also those who saw no reason why mathematics and even higher mathematics and the mathematical sciences should not be taught to girls. Earlier 18th century mathematical authors such as Maria Agnesi and Mme du Châtelet showed – at least to those willing to observe – that there seemed no biological reason why a woman could not master the most difficult higher geometry. It was to take the better part of three long centuries for these ideas to be sorted out, however.

Just as the new mechanical and utilitarian natural philosophy made 'science' more public, so the capacity to appreciate and even use its fruits spread to a wider public. This implied a wider and wider diffusion of education in higher mathematics (which was essentially the algebraic-geometry of conic sections and then the new infinitesimal calculus of curves). The infinitesimal calculus, the so-called geometry-of-the-infinite, as it was to be through the 18$^{th}$ century, was seen as the greatest triumph of the importance of the study of geometry. It revealed the deepest secrets of nature. The teaching of this geometry of the infinite was – from the Jesuits to lay schools and colleges – the ultimate exercise of the limits of the mind's capacity to reason and reason abstractly, and even to grapple with the infinite itself.. It might take one to the mysteries of the infinite and therefore the deepest mysteries of the nature of the world and the contemplation of the mystery of God's infinite nature. Or it might give one the intellectual weight-lifting power to grapple with the abstract relationships that govern an ultimately unknowable world, where we can only operate the abstract relationships amongst phenomena without ever penetrating their ultimate nature. The wonderful thing was that for those who saw education as taking you nearer God, geometry was something of a royal road. And for those who saw education as a security against the arbitrary and non-rational excesses of occult and mystical fantasies, or the arbitrarity of religion, geometry was the surest buttress.

In the very late 17$^{th}$ and 18$^{th}$ centuries the triumph of the mechanical philosophy of the scientific revolution and Newtonian physics led to another interesting *locus geometricus*, where geometry became a social model. One of the most evident characteristics of the English and French Enlightenment is the way that they espoused Newtonianism as practically a new vision of the world and society. Man was a social animal governed by natural laws, and so underneath society were deterministic, rational, natural laws. It was upon these natural laws that the good governance of society needed to based, not the caprices of absolutist monarchs. Just as Newton had demonstrated a rational, well-ordered, and well governed universe, so Enlightenment thinkers argued for a rational, well-ordered and well-governed society. The geometrical good governance of the universe (by natural laws set up by a rational and geometrical God, if you wished to invoke the divine, or a geometrically obedient rational nature if you wanted to be agnostic about it) was a model for the geometrical-like rational good governance of society. The absolute rule of (geometrically demonstrated) law in nature was therefore the reason why society would best be governed in the closest harmony with the natural law of man as a social animal, and so in as close approximation to a form open to reason (as geometry is open to reason) as can be achieved. Amongst French *philosophes* this good governance mean an end to both the arbitrary tyranny of the King and court, and an end to the extra-rational tyranny of the Church. One can only regret that in the end, what replaced them in the French Revolution was even less rational and even less obedient to a reasoned good-governance of society.

However, enlightenment rationalism and determinism did not only end here, with the guillotine, the Terror, and Napoleon. Another expression of the Enlightenment's Newtonian vision of an ultimately rational, geometrical, and understandable or rational universe was that the organisation of human social intercourse in society followed the same principles as this rational and well governed universe, and thus since man was subject to natural social law, so society was best governed in harmony with those social laws. So when in the Declaration of Independence Thomas Jefferson wrote that the American people needed to "assume among the powers of the earth, the separate and equal station to which the Laws of Nature and of Nature's God entitle them", and that certain social laws and rights to be *truths* that are *self-evident*, and that the right and natural form of governance would respect those social laws, they were echoing – in a distant way – that same ultimate faith in a rational and geometrically well ordered universe. We have gone from geometry getting you to absolutist Philosopher-Kings, all the way to geometry getting you to a reasoned and rational democracy. Plato, I'm sure, must have been amused.

I shall stop here … not because the end of the Enlightenment is any sort of an end to the story, but only because to tell the story of the social and scientific role of geometry, its uses and its justifications over the *next* two centuries is at least as long a story as I have already sketched. And anyway, I think my point is already clear; that geometry has occupied any number of different social, intellectual, philosophical, and scientific positions over the ages. But through all these remarkable vicissitudes, one thing remains constant, and that is geometry as the paradigm of truth and certainty, and geometry as the ultimate exercise in clear, logical thinking and reasoning. Whatever else we have found in geometry over the last two and a half thousand years, we have always seen in it a training in clear and abstract thinking. Which is only what Plato said.

### *And so, to end …*

Let me make a penultimate point. Laugh not at the strangeness of the past, lest you be laughed at in turn. For all that many times the role and the justification for the study of geometry in the past seems alien and strange, or downright wrong to us, remember that the actors at the time were no less clever – or silly – than we are today, and that these arguments seemed sensible and reasonable to them at that time and in their context.

Remember that one day, perhaps 500 years from now, someone will be standing in "these ancient and hallowed halls" of the Centre for Mathematical Sciences, this famous stage of so much mathematics, Fields Medals, and the like over the previous half millennium … telling that future audience the story of past mathematics and the singularly odd, peculiar, and even laughable views about mathematics and geometry and its role in our understanding of the world that their predecessors had held here a half millennium ago … but how never-the-less, these strange views still led to the construction of these buildings, and so the place where so much great mathematics has been done.

And what is the moral of my story today? Many things. That the status of geometry is what we make it to be, not ordained by some Divine power; that this timeless and utterly pure form of knowledge exists within the most human of cultural contests; that geometry – so ultra-mundane – is somehow also the most applied of subjects. Since the ancient Greeks geometry has been the paradigm of truth and ordered knowledge, of clear thinking and the rigour of absolute precision of thought. But it's real power is that it is *also* about the world around us. No matter how much geometers try to wriggle out if it, it seems that one of the most fundamental of human scientific intuitions is that the physical world is ultimately geometrical, and that to study geometry is in some sense to uncover some kind of ultimate essence of the physical world around us, and so something essential in what we take to mean *to know*. It would be nice if we could put up over the entrance to the CMS "let no one ignorant of geometry enter here", but perhaps with an *Institute of geometry*, we go one better, reminding mathematicians how special geometry is, and ensuring there is ever more geometry to know.

Geometry is more than just a set of pure theorems that sits somewhere in some divine mind or perfect realm of Forms, awaiting our altruistic and pure investigations. Life, on the large scale, is just a lot more complicated that that. Geometry sits in a social, intellectual, and institutional context, and in some practical sense, that context is as much a part of the geometry of any community or epoch as are the theorems. Geometry needs a culture and a society to sustain it, for one reason or another, and to support those who teach and research into its frontiers. Those who build buildings and so make places for that teaching and research sit as part of a tradition in the support of geometry that goes back at least to the ancient Greeks, and the very foundations of Western, scientific, liberal, intellectual, society. As much as the philosophical context and our scientific curiosity, the bricks and mortar of our buildings help to made and to sustain geometry, and so to sustain one of the most constant elements of the foundations of our culture and our world. Thank you, Dr. Dill Faulkes, for our *Institute of Geometry*, for the bricks and the mortar, and therefore for the theorems to come.

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