

L I N E A R

L O G I C

# CLASSICAL LINEAR LOGIC

- Separates out the complexities arising from the rules of weakening, contraction in classical sequent calculus

$$\frac{\Gamma \vdash \Delta}{\Gamma, A \vdash \Delta}$$

$$\frac{\Gamma \vdash \Delta}{\Gamma \vdash \Delta, B}$$

$$\frac{\Gamma, A, A \vdash \Delta}{\Gamma, A \vdash \Delta}$$

$$\frac{\Gamma \vdash \Delta, B, B}{\Gamma \vdash \Delta, B}$$

- Classical in the sense that it maintains the symmetry of classical sequent calculus.

# RULES FOR CONJUNCTION (and DISJUNCTION)

Conjunction

$$\frac{\Gamma \vdash A, \Delta \quad \Gamma' \vdash B, \Delta'}{\Gamma, \Gamma' \vdash A \wedge B, \Delta, \Delta'}$$
$$\frac{\Gamma, A, B \vdash \Delta}{\Gamma, A \wedge B \vdash \Delta}$$

} xve form

$$\frac{\Gamma \vdash A, \Delta \quad \Gamma \vdash B, \Delta}{\Gamma \vdash A \wedge B, \Delta}$$
$$\frac{\Gamma, A \vdash \Delta}{\Gamma, A \wedge B \vdash \Delta} \quad \frac{\Gamma, B \vdash \Delta}{\Gamma, A \wedge B \vdash \Delta}$$

} +ve form

Disjunction dual to the above

# SYNTAX of CLL

(Propositional fragment only)

## Multiplicatives

Conjunctions  $I, \otimes$

Disjunctions  $\perp, \wp$

## Exponentials ('Modalities')

"Of course"  $!$  "Potentially infinitary tensor"

"Why not"  $?$  "Potentially infinitary par"

## Additives

Conjunctions  $T, \&$

Disjunctions  $0, \oplus$

## Involutions

Negation  $\neg$

$( )^\perp$

# BASIC RULES

AXIOM

$$\frac{}{A \vdash A}$$

CUT

$$\frac{\Gamma \vdash \Delta, A \quad A, \Gamma' \vdash \Delta'}{\Gamma, \Gamma' \vdash \Delta, \Delta'}$$

EXCHANGE

$$\frac{\Gamma, A, B, \Gamma' \vdash \Delta}{\Gamma, B, A, \Gamma' \vdash \Delta}$$

$$\frac{\Gamma \vdash \Delta, C, D, \Delta'}{\Gamma \vdash \Delta, D, C, \Delta'}$$

NEGATION

$$\frac{\Gamma, A \vdash \Delta}{\Gamma \vdash A^{\perp}, \Delta}$$

$$\frac{\Gamma \vdash C, \Delta}{\Gamma, C^{\perp} \vdash \Delta}$$

WNTΓLIGICVLINE

BNPEZ

# MULTIPLICATIVE RULES

## Conjunctions

$$\frac{}{\vdash I}$$

$$\frac{\Gamma \vdash \Delta}{\Gamma, I \vdash \Delta}$$

$$\frac{\Gamma \vdash A, \Delta \quad \Gamma' \vdash B, \Delta'}{\Gamma, \Gamma' \vdash A \otimes B, \Delta, \Delta'}$$

$$\frac{\Gamma, A, B \vdash \Delta}{\Gamma, A \otimes B \vdash \Delta}$$

## Disjunctions

$$\frac{\Gamma \vdash \Delta}{\Gamma \vdash \perp, \Delta}$$

$$\frac{}{\perp \vdash}$$

$$\frac{\Gamma \vdash C, D, \Delta}{\Gamma \vdash C \wp D, \Delta}$$

$$\frac{\Gamma, C \vdash \Delta \quad \Gamma', D \vdash \Delta'}{\Gamma, \Gamma', C \wp D \vdash \Delta, \Delta'}$$

EXHONENTIALIUS BNGEZ

# EXPONENTIAL RULES

'Of course'

(Comonad rules) 
$$\frac{\Gamma, A \vdash \Delta}{\Gamma, !A \vdash \Delta}$$

$$\frac{! \Gamma \vdash A, ? \Delta}{! \Gamma \vdash !A, ? \Delta}$$

(Comonoid rules) 
$$\frac{\Gamma \vdash \Delta}{\Gamma, !A \vdash \Delta}$$

$$\frac{\Gamma, !A, !A \vdash \Delta}{\Gamma, !A \vdash \Delta}$$

'Why not'

(par monoid rules) 
$$\frac{\Gamma \vdash C, \Delta}{\Gamma \vdash ?C, \Delta}$$

$$\frac{! \Gamma, C \vdash ? \Delta}{! \Gamma, ?C \vdash ? \Delta}$$

(par comonoid rules) 
$$\frac{\Gamma \vdash \Delta}{\Gamma \vdash ?C, \Delta}$$

$$\frac{\Gamma \vdash ?C, ?C, \Delta}{\Gamma \vdash ?C, \Delta}$$

ADDITIONAL

# ADDITIVES

Skip the rules !!! It should be enough to know that eventually they give proof theoretic equivalences

$$!T \cong I$$

$$!(A \times B) \cong !A \otimes !B$$

$$?0 \cong \perp$$

$$?(A \oplus B) \cong ?A \wp ?B$$

EXAMPLE



# EXAMPLE

(Weak distributivity)

$$\begin{array}{c} \frac{A \vdash A \quad B \vdash B}{A, B \vdash A \otimes B} \quad C \vdash C \\ \hline A, B \wp C \vdash A \otimes B, C \\ \hline A, B \wp C \vdash (A \otimes B) \wp C \\ \hline A \otimes (B \wp C) \vdash (A \otimes B) \wp C \end{array}$$

NOTE Two 'irrelevant' choices

⇓  
Proof Nets

NOTE While  $A \otimes (B \wp C) \vdash (A \otimes B) \wp C$  is canonically provable, there can be no cut-free (& hence no) proof of  $(A \otimes B) \wp C \vdash A \otimes (B \wp C)$

# A MANY-FEATURED PROOF

$$\begin{array}{l}
 \underline{A \vdash A} \\
 \underline{!A \vdash A} \\
 \underline{!A, A^{\perp} \vdash} \\
 \underline{!A, ?A^{\perp} \vdash} \quad B \vdash B \\
 \underline{!A, (?A^{\perp} \wp B) \vdash B} \\
 \underline{!A, !(?A^{\perp} \wp B) \vdash B} \\
 \underline{!A, !(?A^{\perp} \wp B) \vdash !B}
 \end{array}$$

$$\begin{array}{l}
 \underline{A \vdash A} \\
 \underline{!A \vdash A} \\
 \underline{!A, A^{\perp} \vdash} \\
 \underline{!A, ?A^{\perp} \vdash} \quad ?B^{\perp} \wp C \vdash ?B^{\perp} \wp C \\
 \underline{!A, (?A^{\perp} \wp (?B^{\perp} \wp C)) \vdash ?B^{\perp} \wp C} \\
 \underline{!A, !(?A^{\perp} \wp (?B^{\perp} \wp C)) \vdash ?B^{\perp} \wp C} \\
 \underline{!A, !(?A^{\perp} \wp (?B^{\perp} \wp C)) \vdash !(?B^{\perp} \wp C)}
 \end{array}$$

$$\begin{array}{l}
 \underline{B \vdash B} \\
 \underline{!B \vdash B} \\
 \underline{!B, B^{\perp} \vdash} \\
 \underline{!B, ?B^{\perp} \vdash} \quad C \vdash C \\
 \underline{!B, (?B^{\perp} \wp C) \vdash C} \\
 \underline{!B, !(?B^{\perp} \wp C) \vdash C}
 \end{array}$$

$$\underline{!A, !(?A^{\perp} \wp (?B^{\perp} \wp C)), !B \vdash C}$$

$$\underline{!A, !A, !(?A^{\perp} \wp B), !(?A^{\perp} \wp (?B^{\perp} \wp C)) \vdash C}$$

$$!A, !(?A^{\perp} \wp B), !(?A^{\perp} \wp (?B^{\perp} \wp C)) \vdash C$$

$$\underline{A, A \rightarrow (B \rightarrow C) \vdash B \rightarrow C}$$

$$\underline{B, B \rightarrow C \vdash C}$$

$$\underline{A, (A \rightarrow B) \vdash B}$$

$$\underline{A, A \rightarrow (B \rightarrow C), B \vdash C}$$

$$A, (A \rightarrow B) \quad A \rightarrow (B \rightarrow C) \vdash C$$

# SOME DUALITIES

$$\begin{array}{c}
 \frac{A \vdash A}{\vdash A^+, A} \\
 \frac{B \vdash B}{\vdash B^+, B} \\
 \hline
 \vdash A^+, B^+, (A \otimes B) \\
 \hline
 (A \otimes B)^\perp \vdash A^+, B^+ \\
 \hline
 (A \otimes B)^\perp \vdash A^+ \wp B^+
 \end{array}$$

$$\begin{array}{c}
 \frac{A \vdash A}{A, A^+ \vdash} \\
 \frac{B \vdash B}{B, B^+ \vdash} \\
 \hline
 A, B, A^+ \wp B^+ \vdash \\
 \hline
 A \otimes B, A^+ \wp B^+ \vdash \\
 \hline
 A^+ \wp B^+ \vdash (A \otimes B)^\perp
 \end{array}$$

"So"  $(A \otimes B)^\perp \cong A^+ \wp B^+$

$$\begin{array}{c}
 \frac{A \vdash A}{\vdash A, A^+} \\
 \frac{\vdash A, ?A^+}{\vdash !A, ?A^+} \\
 \hline
 (!A)^\perp \vdash ?A^+
 \end{array}$$

$$\begin{array}{c}
 \frac{A \vdash A}{A, A^+ \vdash} \\
 \frac{A, A^+ \vdash}{!A, A^+ \vdash} \\
 \frac{!A, A^+ \vdash}{!A, ?A^+ \vdash} \\
 \hline
 ?A^+ \vdash (!A)^\perp
 \end{array}$$

"So"  $(!A)^\perp \cong ?A^+$

# ONE-SIDED CALCULUS

(Modulo dualities)

Axiom

$$\frac{}{\vdash A, A^+}$$

Cut

$$\frac{\vdash A, \Gamma \quad \vdash A^+, \Delta}{\vdash \Gamma, \Delta}$$

Exchange

$$\frac{\vdash \Gamma, A, B, \Delta}{\vdash \Gamma, B, A, \Delta}$$

Multiplicatives

$$\frac{}{\vdash I}$$

$$\frac{\vdash \Gamma}{\vdash \perp, \Gamma}$$

$$\frac{\vdash A, \Gamma \quad \vdash B, \Delta}{\vdash A \otimes B, \Gamma, \Delta}$$

$$\frac{\vdash A, B, \Gamma}{\vdash A \wp B, \Gamma}$$

symmetry of  $I \perp \Gamma$

Exponentials

$$\frac{\vdash A, \Gamma}{\vdash ?A, \Gamma}$$

$$\frac{\vdash A, ?\Delta}{\vdash !A, ?\Delta}$$

$$\frac{\vdash \Gamma}{\vdash ?A \Gamma}$$

$$\frac{\vdash ?A, ?A, \Delta}{\vdash ?A, \Delta}$$

Additives — no room!

INTUITIONISTIC LINEAR LOGIC

# INTUITIONISTIC LINEAR LOGIC

- Asymmetric system
- Good for many purposes  
(eg. foundations of functional programming)
- lots of naturally occurring 'models'.

## Syntax of ILL

Multiplicatives  $I, \otimes, \multimap$

Exponential  $!$

Additives  $\top, \&$

USES: Single conclusion sequents

SEMANTIC MODELS for ILL

# SEQUENT RULES for ILL

Axiom

$$\frac{}{A \vdash A}$$

Cut

$$\frac{\Gamma \vdash A \quad A, \Delta \vdash B}{\Gamma, \Delta \vdash B}$$

Exchange

$$\frac{\Gamma, A, B, \Delta \vdash C}{\Gamma, B, A, \Delta \vdash C}$$

Multiplicatives

$$\frac{}{\Gamma \vdash I}$$

$$\frac{\Gamma \vdash A}{\Gamma, I \vdash A}$$

$$\frac{\Gamma \vdash A \quad \Delta \vdash B}{\Gamma, \Delta \vdash A \otimes B}$$

$$\frac{\Gamma, A, B \vdash C}{\Gamma, A \otimes B \vdash C}$$

$$\frac{\Gamma, A \vdash B}{\Gamma \vdash A \multimap B}$$

$$\frac{\Gamma \vdash A \quad \Delta, B \vdash C}{\Gamma, \Delta, A \multimap B \vdash C}$$

Exponential

$$\frac{\Gamma, A \vdash B}{\Gamma, !A \vdash B}$$

$$\frac{! \Gamma \vdash A}{! \Gamma \vdash !A}$$

$$\frac{\Gamma \vdash B}{\Gamma, !A \vdash B}$$

$$\frac{\Gamma, !A, !A \vdash B}{\Gamma, !A \vdash B}$$

Additives

again no room

no room for

no room for

# GIRARD TRANSLATION OF INTUITIONISTIC LOGIC (very good for minimal logic)

Intuitionistic logic ILL

$$A \longmapsto t(A)$$

ATOMS

$$A \longmapsto A$$

RECURSIVE CLAUSES

$$1 \longmapsto T$$

$$A \wedge B \longmapsto t(A) \& t(B)$$

$$A \rightarrow B \longmapsto !t(A) \multimap t(B)$$

TRANSLATION OF DEDUCTIONS

$$\Gamma \vdash A \longmapsto !t(\Gamma) \vdash t(A)$$

THE TRANSLATION CORRECTNESS OF

# CORRECTNESS OF THE TRANSLATION

Essentially require to show

$$\frac{A \wedge B \vdash C}{A \vdash B \rightarrow C}$$

is reflected by the translation.

But

$$\frac{\frac{\frac{\frac{\frac{! (A \wedge B)^t \vdash C^t}{!(A^t \& B^t) \vdash C^t}}{!A^t \otimes !B^t \vdash C^t}}{!A^t \vdash !B^t \rightarrow C^t}}{!A^t \vdash (B \rightarrow C)^t}}$$

equality

by an isomorphism

by ILL

equality

(as a result of work of [1] [2] [3] [4] [5] [6] [7] [8] [9] [10] [11] [12] [13] [14] [15] [16] [17] [18] [19] [20] [21] [22] [23] [24] [25] [26] [27] [28] [29] [30] [31] [32] [33] [34] [35] [36] [37] [38] [39] [40] [41] [42] [43] [44] [45] [46] [47] [48] [49] [50] [51] [52] [53] [54] [55] [56] [57] [58] [59] [60] [61] [62] [63] [64] [65] [66] [67] [68] [69] [70] [71] [72] [73] [74] [75] [76] [77] [78] [79] [80] [81] [82] [83] [84] [85] [86] [87] [88] [89] [90] [91] [92] [93] [94] [95] [96] [97] [98] [99] [100] [101] [102] [103] [104] [105] [106] [107] [108] [109] [110] [111] [112] [113] [114] [115] [116] [117] [118] [119] [120] [121] [122] [123] [124] [125] [126] [127] [128] [129] [130] [131] [132] [133] [134] [135] [136] [137] [138] [139] [140] [141] [142] [143] [144] [145] [146] [147] [148] [149] [150] [151] [152] [153] [154] [155] [156] [157] [158] [159] [160] [161] [162] [163] [164] [165] [166] [167] [168] [169] [170] [171] [172] [173] [174] [175] [176] [177] [178] [179] [180] [181] [182] [183] [184] [185] [186] [187] [188] [189] [190] [191] [192] [193] [194] [195] [196] [197] [198] [199] [200] [201] [202] [203] [204] [205] [206] [207] [208] [209] [210] [211] [212] [213] [214] [215] [216] [217] [218] [219] [220] [221] [222] [223] [224] [225] [226] [227] [228] [229] [230] [231] [232] [233] [234] [235] [236] [237] [238] [239] [240] [241] [242] [243] [244] [245] [246] [247] [248] [249] [250] [251] [252] [253] [254] [255] [256] [257] [258] [259] [260] [261] [262] [263] [264] [265] [266] [267] [268] [269] [270] [271] [272] [273] [274] [275] [276] [277] [278] [279] [280] [281] [282] [283] [284] [285] [286] [287] [288] [289] [290] [291] [292] [293] [294] [295] [296] [297] [298] [299] [300] [301] [302] [303] [304] [305] [306] [307] [308] [309] [310] [311] [312] [313] [314] [315] [316] [317] [318] [319] [320] [321] [322] [323] [324] [325] [326] [327] [328] [329] [330] [331] [332] [333] [334] [335] [336] [337] [338] [339] [340] [341] [342] [343] [344] [345] [346] [347] [348] [349] [350] [351] [352] [353] [354] [355] [356] [357] [358] [359] [360] [361] [362] [363] [364] [365] [366] [367] [368] [369] [370] [371] [372] [373] [374] [375] [376] [377] [378] [379] [380] [381] [382] [383] [384] [385] [386] [387] [388] [389] [390] [391] [392] [393] [394] [395] [396] [397] [398] [399] [400] [401] [402] [403] [404] [405] [406] [407] [408] [409] [410] [411] [412] [413] [414] [415] [416] [417] [418] [419] [420] [421] [422] [423] [424] [425] [426] [427] [428] [429] [430] [431] [432] [433] [434] [435] [436] [437] [438] [439] [440] [441] [442] [443] [444] [445] [446] [447] [448] [449] [450] [451] [452] [453] [454] [455] [456] [457] [458] [459] [460] [461] [462] [463] [464] [465] [466] [467] [468] [469] [470] [471] [472] [473] [474] [475] [476] [477] [478] [479] [480] [481] [482] [483] [484] [485] [486] [487] [488] [489] [490] [491] [492] [493] [494] [495] [496] [497] [498] [499] [500] [501] [502] [503] [504] [505] [506] [507] [508] [509] [510] [511] [512] [513] [514] [515] [516] [517] [518] [519] [520] [521] [522] [523] [524] [525] [526] [527] [528] [529] [530] [531] [532] [533] [534] [535] [536] [537] [538] [539] [540] [541] [542] [543] [544] [545] [546] [547] [548] [549] [550] [551] [552] [553] [554] [555] [556] [557] [558] [559] [560] [561] [562] [563] [564] [565] [566] [567] [568] [569] [570] [571] [572] [573] [574] [575] [576] [577] [578] [579] [580] [581] [582] [583] [584] [585] [586] [587] [588] [589] [590] [591] [592] [593] [594] [595] [596] [597] [598] [599] [600] [601] [602] [603] [604] [605] [606] [607] [608] [609] [610] [611] [612] [613] [614] [615] [616] [617] [618] [619] [620] [621] [622] [623] [624] [625] [626] [627] [628] [629] [630] [631] [632] [633] [634] [635] [636] [637] [638] [639] [640] [641] [642] [643] [644] [645] [646] [647] [648] [649] [650] [651] [652] [653] [654] [655] [656] [657] [658] [659] [660] [661] [662] [663] [664] [665] [666] [667] [668] [669] [670] [671] [672] [673] [674] [675] [676] [677] [678] [679] [680] [681] [682] [683] [684] [685] [686] [687] [688] [689] [690] [691] [692] [693] [694] [695] [696] [697] [698] [699] [700] [701] [702] [703] [704] [705] [706] [707] [708] [709] [710] [711] [712] [713] [714] [715] [716] [717] [718] [719] [720] [721] [722] [723] [724] [725] [726] [727] [728] [729] [730] [731] [732] [733] [734] [735] [736] [737] [738] [739] [740] [741] [742] [743] [744] [745] [746] [747] [748] [749] [750] [751] [752] [753] [754] [755] [756] [757] [758] [759] [760] [761] [762] [763] [764] [765] [766] [767] [768] [769] [770] [771] [772] [773] [774] [775] [776] [777] [778] [779] [780] [781] [782] [783] [784] [785] [786] [787] [788] [789] [790] [791] [792] [793] [794] [795] [796] [797] [798] [799] [800] [801] [802] [803] [804] [805] [806] [807] [808] [809] [810] [811] [812] [813] [814] [815] [816] [817] [818] [819] [820] [821] [822] [823] [824] [825] [826] [827] [828] [829] [830] [831] [832] [833] [834] [835] [836] [837] [838] [839] [840] [841] [842] [843] [844] [845] [846] [847] [848] [849] [850] [851] [852] [853] [854] [855] [856] [857] [858] [859] [860] [861] [862] [863] [864] [865] [866] [867] [868] [869] [870] [871] [872] [873] [874] [875] [876] [877] [878] [879] [880] [881] [882] [883] [884] [885] [886] [887] [888] [889] [890] [891] [892] [893] [894] [895] [896] [897] [898] [899] [900] [901] [902] [903] [904] [905] [906] [907] [908] [909] [910] [911] [912] [913] [914] [915] [916] [917] [918] [919] [920] [921] [922] [923] [924] [925] [926] [927] [928] [929] [930] [931] [932] [933] [934] [935] [936] [937] [938] [939] [940] [941] [942] [943] [944] [945] [946] [947] [948] [949] [950] [951] [952] [953] [954] [955] [956] [957] [958] [959] [960] [961] [962] [963] [964] [965] [966] [967] [968] [969] [970] [971] [972] [973] [974] [975] [976] [977] [978] [979] [980] [981] [982] [983] [984] [985] [986] [987] [988] [989] [990] [991] [992] [993] [994] [995] [996] [997] [998] [999] [1000])

BIG DISCOVERY



# BIG DISCOVERY

(especially in view of LC, LU, ...)

More usual logic can be decomposed into logic-like elements which correspond to the 'propositions' of linear logic.

- How to think of these 'micro-propositions'?
- In what way to think of the world of propositions as standing within this wider world?

The idea does (computer science) work.

Does it make philosophical sense?

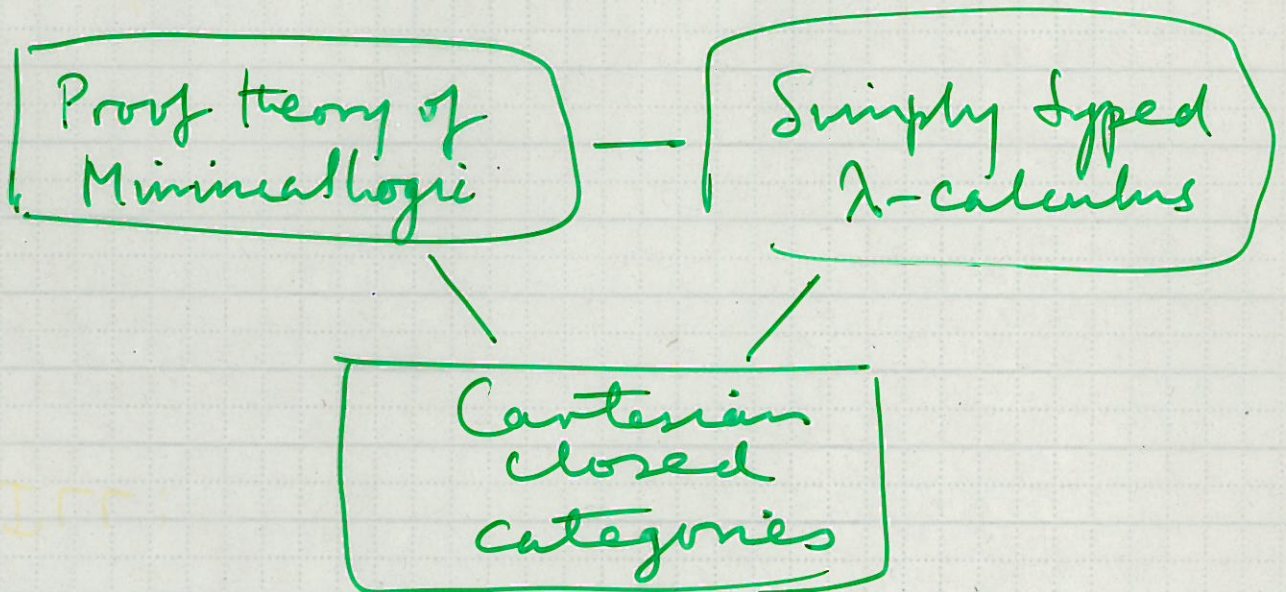
# MODELS

via

## CURRY - HOWARD

(i.e. after identification of cut-free proofs)

### COMPARE



For IFF:

EM CFF:

CATEGORICAL MODELS OF CFF/IFF

# CATEGORICAL MODELS OF CLL/ILL

For CLL:

Have a  $*$ -autonomous category plus  
a comonad s.t.  $\dashv\dashv$  (plus products).

For ILL:

Have a symmetric monoidal closed  
category plus a comonad s.t.  $\dashv\dashv$   
(plus products).

## Examples

CLL      V-lattices, coherence spaces

ILL      Vector spaces, domains & linear maps

(Also assorted categories of  
games)

# DUALITY CONSTRUCTIONS

(An example)

Suppose  $\mathcal{C}$  is smcc with products;

then  $\mathcal{C} \times \mathcal{C}^{\text{op}}$  is  $*$ -autonomous.

Enough to define  $I, \otimes, (\ )^\perp$   
on  $\mathcal{C} \times \mathcal{C}^{\text{op}}$  & check it works:-

$I$  is  $(I, \tau)$

$$(u, x) \otimes (v, y) = (u \otimes v, (v \multimap x) \times (u \multimap y))$$

$$(u, x)^\perp = (x, u)$$

REPRESENTATION OF PROOFS

# REPRESENTATION OF PROOFS

SEEK Canonical representation of  
(equivalence classes of) normal proofs



CONCRETE presentation of the free  
 $\mathcal{S}$  &  $\mathcal{S}$  category

(Maybe good for applications.)

Suitable such for linear logic are

'Proof Nets'

PROOF NETS for CLL

# PROOF NETS for CLL

(Multiplicative fragment)

Graph with links as follows :-

Axiom 

Cut 

Tensor 

Par 

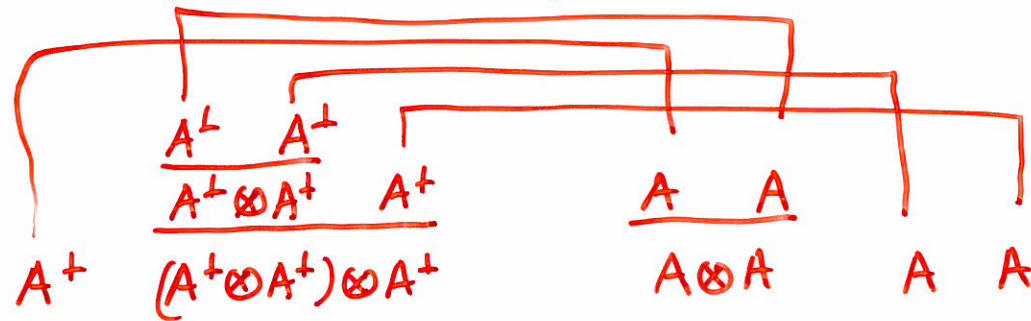
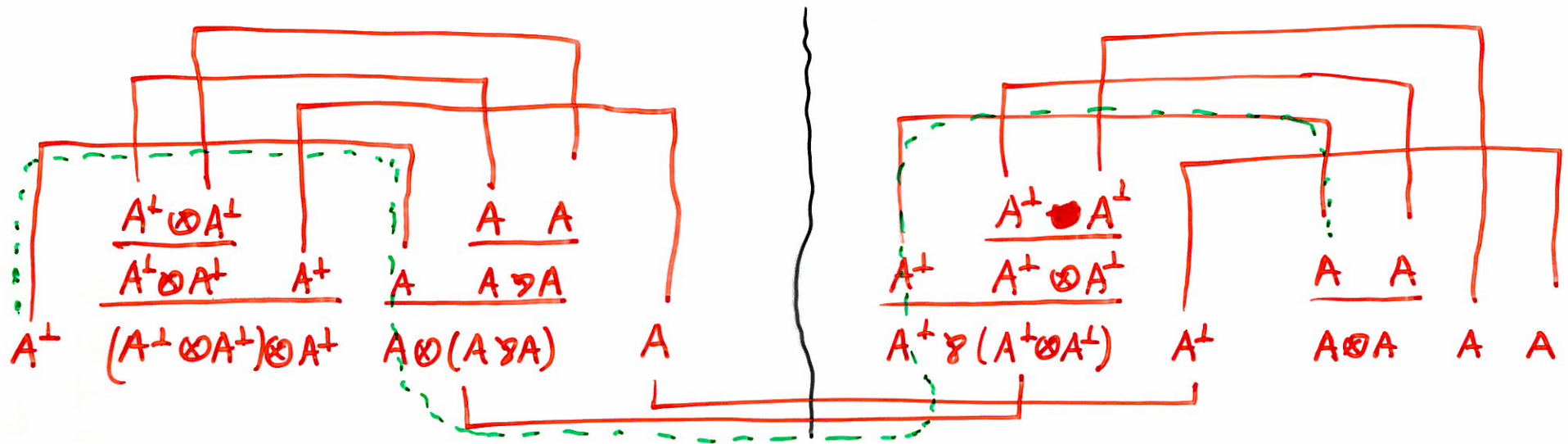


A switching chooses one or other of these

**CORRECTNESS CRITERION** A graph (without cut) represents (an equivalence of) / a proof iff for every switching (Danos - Regnier) it is a 'tree' i.e. it is connected acyclic.

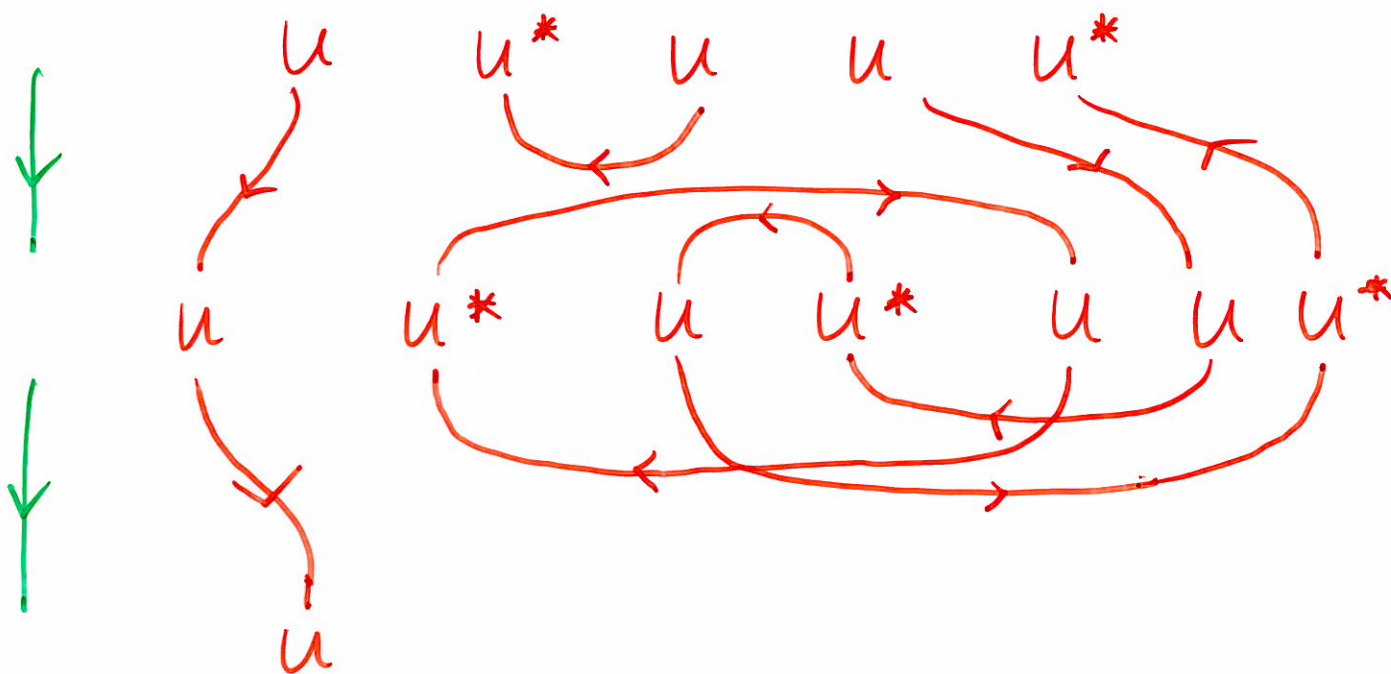
# 'TURBO' CUT ELIMINATION

(Connect the wires!)



# THE FREE 'COMPACT CLOSED' CATEGORY

(Degenerate case when  $\otimes = \wp$ )



i.e. linear logic with no correctness criterion



# FULL COMPLETENESS THMS

Assume a  $*$ -autonomous  $\mathbb{C}$ .

If  $\Gamma(A_1, \dots, A_n)$  is the  $x$ -ve fragment, then for each interpretation  $\llbracket A_i \rrbracket \in \mathbb{C}$ , we have  $\llbracket \Gamma(A_1, \dots, A_n) \rrbracket \in \mathbb{C}$ .

A proof  $\vdash \Gamma(A_1, \dots, A_n)$  in CLL gives a uniform map

$$p_{\llbracket \cdot \rrbracket} : I \longrightarrow \llbracket \Gamma(A_1, \dots, A_n) \rrbracket \in \mathbb{C}$$

QUESTION Do such (non-trivial) uniform maps correspond exactly to proofs?

# "SCHUR-WEYL DUALITY"

(Generalizes: Which matrices commute with all matrices?)

Let  $V$  be a finite dimensional vector space of dimension  $\geq 2$ , and  $\mathbb{C}$  the compact closed subcategory of finite dimensional vector spaces generated by  $V$ .

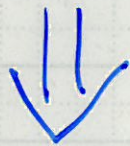
Now restrict to  $\bar{\mathbb{C}}$  whose maps are uniform w.r.t. the action of  $GL(V)$ .

Then  $\bar{\mathbb{C}}$  is the free  $k$ -linear compact closed category on a single object.

# ABSTRACT GEOMETRY OF INTERACTION

Based on (certain) symmetric monoidal categories with trace

Trace = Feedback loop



(Special) compact closed categories.

Interpret CLL in degenerate models so

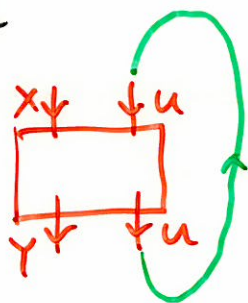
Cut  $\leftarrow$   $\rightarrow$  Contraction of indices

Extract computational information!

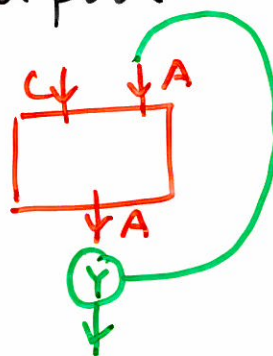
# CARTESIAN CATEGORIES WITH TRACE

For a category with  $\times$ s, a trace is equivalent to a (good ish) fixed point operator.

Trace



Fixed points

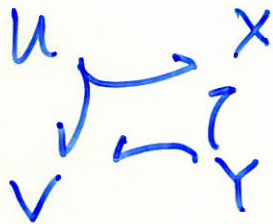


# TOY EXAMPLE

Objects Pairs of (finite) sets  
 $(U, X)$

Maps  $(U, X) \longrightarrow (V, Y)$

Partial maps



$u \longrightarrow X+V$   
 $Y \longrightarrow X+V$

Composition

