Dynamical Systems (M24)

Gabriel P. Paternain

Dynamical Systems is the study of the long term behaviour of systems that depend on time. The evolution of a particular point is called an *orbit* and the theory attempts to describe the orbit structure complexity of a system. The modern theory of dynamical systems originated at the end of the 19th century trying to answer fundamental questions in celestial mechanics (is the solar system stable?) and nowadays constitutes an important branch of Mathematics with far reaching applications.

Contents:

1. The notion of a dynamical system. Examples and basic notions. Suspensions and cross sections.
2. Topological dynamics. Limits sets and recurrence, topological transitivity and topological mixing. Expansiveness. Topological entropy.
3. Symbolic dynamics. Subshifts and codes, subshifts of finite type, the Perron-Frobenius theorem. Topological entropy and the zeta function of a subshift of finite type.
4. Ergodic theory. Recurrence, ergodicity and mixing. Ergodic theorems. Invariant measures for continuous maps. Unique ergodicity.
5. Hyperbolic dynamics. Hyperbolic sets, invariant cones. Stability of hyperbolic sets. Stable and unstable manifolds. Horseshoes and transverse homoclinic points. Local product structure and locally maximal hyperbolic sets. Anosov diffeomorphisms, Axiom A and structural stability. Geodesic flows of negatively curved manifolds.

Desirable Previous Knowledge

It might help to have some knowledge of manifolds, flows and vector fields, but it is not strictly necessary. You will learn all that is needed in the first couple of weeks of the *Differential Geometry* course offered in the Michaelmas term.

Introductory Reading

1. M. Brin, G. Stuck, *Introduction to Dynamical Systems,* Cambridge University Press, 2002.
2. R.L. Devaney, *An introduction to chaotic dynamical systems,* Addison-Wesley, 1989.
3. B. Hasselblatt, A. Katok, *A first course in dynamics. With a panorama of recent developments,* Cambridge University Press, New York, 2003.

Reading to complement course material

1. M. Brin, G. Stuck, *Introduction to Dynamical Systems,* Cambridge University Press, 2002.
2. C. Robinson, *Dynamical Systems,* Studies in Advanced Mathematics, CRC Press, Boca Raton, 1995.
3. A. Katok, B. Hasselblatt, *Introduction to the modern theory of dynamical systems,* Encyclopedia of Mathematics and its applications **54**, Cambridge University Press, 1995.
4. J. Palis and W. de Melo, *Geometric theory of dynamical systems,* Springer-Verlag, New York, 1982.
5. P. Walters, *An introduction to ergodic theory,* Graduate Texts in Mathematics, Springer-Verlag, 1982.

There will be 3 example sheets and 3 example classes given by the lecturer.