Differential Geometry (M24)

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This course aims to provide an introduction to Differential Geometry.

Contents

We will try to cover the following topics (not necessarily in this order):

1. *Differentiable Manifolds.* Definition and examples. Tangent vectors, tangent and cotangent bundles. Smooth maps and the inverse function theorem. Differential forms, Stokes' theorem and de Rham cohomology.

2. *Vector bundles.* Structure group, principal bundles. Connections and curvature.

3. *Riemannian geometry.* Riemannian metrics, Levi-Civita connection. Geodesics, exponential map and Gauss' lemma. The Riemann curvature tensor, sectional curvature, Ricci curvature and scalar curvature. The Hodge star operator and the Laplace-Beltrami operator.

Desirable Previous Knowledge

Familiarity with the classical theory of curves and surfaces will be useful.

References

1. I. Chavel, *Riemannian geometry—a modern introduction,* Cambridge Tracts in Mathematics, 108. Cambridge University Press, 1993.

2. S. Gallot, D. Hulin,J. Lafontaine, *Riemannian geometry,* Universitext, Springer-Verlag, Berlin, 2004.

3. V. Guillemin, A. Pollack, *Differential topology,* Prentice-Hall, Inc., Englewood Cliffs, N.J., 1974.

4. J. Jost, *Riemannian geometry and geometric analysis,* Universitext, Springer-Verlag, Berlin, 2005

5. F.W. Warner, *Foundations of differentiable manifolds and Lie groups,* Graduate Texts in Mathematics, 94. Springer-Verlag, New York-Berlin, 1983.