SOLUTION OF EXERCISE 7

CAUCHER BIRKAR (C.BIRKAR@DPMMS.CAM.AC.UK)

I leave the proof of the fact that $d_x \colon m_x \to T^*_{X,x}$ is well-defined to the students (as it is quite straightforward).

Let I_x be the maximal ideal of $k[X]$ corresponding to $x \in X$. We proved in the lectures that we have an isomorphism $I_x/I_x^2 \to T^*_{X,x}$ of k-vector spaces induced by a differential map $d_x \colon I_x \to T^*_{X,x}$. Since $\mathcal{O}_{X,x}$ is the localisation of $k[X]$ at I_x we have a k-linear map $\alpha \colon I_x/I_x^2 \to m_x/m_x^2$ so we get a commutative diagram

$$\begin{array}{ccc} I_x/I_x^2 & \xrightarrow{\alpha} & m_x/m_x^2 \\ & \searrow & \downarrow \\ & & T^*_{X,x} \end{array}$$

Now each element ω of m_x/m_x^2 can be written as $\sum_i a_i(t_i - x_i)$ where $x = (x_1, \dots, x_n)$ and $a_i \in k$. But ω can also be considered as an element of I_x/I_x^2 hence α is surjective. Since $I_x/I_x^2 \to T^*_{X,x}$ is an isomorphism it is obvious that $m_x/m_x^2 \to T^*_{X,x}$ should also be an isomorphism.

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