Review of some basic linear analysis

The following is the content of a typical first course in Functional Analysis like the Part II Linear Analysis course here in Cambridge. I will take these topics for granted and assume that you are familiar with them. Having said that, I will occasionally recall definitions and theorems in the lectures.

- Definition of normed space and Banach space.
- Examples: ℓ_p^n, ℓ_p, c_0, $L_p(\mu)$, $C(K)$, Hilbert spaces.
- Topology and geometry of a normed space X; the unit ball B_X and the unit sphere S_X.
- Operators, *i.e.,* bounded linear maps, between Banach spaces; operator norm; the space $\mathcal{B}(X,Y)$ of all operators from X to Y.
- Isomorphisms; isomorphic embeddings; isometric isomorphisms.
- Bounded linear functionals; dual space.
- Inequalities of Hölder and Minkowski; the dual of ℓ_p and c_0.
- Finite-dimensional spaces; equivalence of norms; Riesz's lemma; characterization of finite-dimensionality in terms of compactness of the unit ball.
- Baire Category Theorem; Principle of Uniform Boundedness; Banach–Steinhaus; Open Mapping Lemma; Open Mapping Theorem; Closed Graph Theorem.
- Spaces of continuous functions on compact (or locally compact) Hausdorff spaces; Urysohn's lemma; Tietze's extension theorem; Stone–Weierstrass Theorem; Arzelà–Ascoli.
- Inner product spaces; Hilbert space; orthonormal systems; Bessel's inequality; Parseval's theorem; adjoint operators; spectral theory.