

Functional Analysis (M24)

A Zsák

This course covers many of the major theorems of abstract Functional Analysis. It is intended to provide a foundation for several areas of pure and applied mathematics. We will cover the following topics:

Hahn–Banach Theorems on the extension of linear functionals. Locally convex spaces.

Duals of the spaces $L_p(\mu)$ and $C(K)$. The Radon–Nikodym Theorem and the Riesz Representation Theorem.

Weak and weak-* topologies. Theorems of Mazur, Goldstine, Banach–Alaoglu. Reflexivity and local reflexivity.

Hahn–Banach Theorems on separation of convex sets. Extreme points and the Krein–Milman theorem. Partial converse and the Banach–Stone Theorem.

Banach algebras, elementary spectral theory. Commutative Banach algebras and the Gelfand representation theorem. Holomorphic functional calculus.

Hilbert space operators, C^* -algebras. The Gelfand–Naimark theorem. Spectral theorem for commutative C^* -algebras. Spectral theorem and Borel functional calculus for normal operators.

Some additional topics time permitting. For example, uniform convexity and smoothness, ultra-products, the Fréchet–Kolmogorov Theorem, weakly compact subsets of $L_1(\mu)$, the Eberlein–Šmulian and the Krein–Šmulian theorems, the Gelfand–Naimark–Segal construction.

Prerequisites

Thorough grounding in basic topology and analysis. Some knowledge of basic functional analysis and basic measure theory (much of which will be recalled either in lectures or via handouts). In Spectral Theory we will make use of basic complex analysis. For example, Cauchy’s Theorem, Cauchy’s Integral Formula and the Maximum Modulus Principle.

Literature

1. Allan, Graham R. *Introduction to Banach spaces and algebras (prepared for publication by H. Garth Dales)*. Oxford University Press, 2011.
2. Bollobás, Béla *Linear analysis: an introductory course*. Cambridge University Press, 1990.
3. Murphy, Gerard J. *C^* -Algebras and Operator Theory*. Academic Press, Inc., 1990.
4. Rudin, Walter *Real & Complex Analysis*. McGraw-Hill, 1987.
5. Rudin, Walter *Functional Analysis*. McGraw-Hill, 1990.
6. Taylor, S. J. *Introduction to measure and integration*. Cambridge University Press 1973.

Additional support

Four examples sheets will be provided and four associated examples classes will be given. There will be a revision class in the Easter Term. There will be some material as well as examples sheets and announcements available at www.dpmms.cam.ac.uk/~az10000/