1. Show that the Empty-Set Axiom is deducible from the Axioms of Infinity and Separation (or, if you prefer, just from the Axiom of Infinity), and that the Axiom of Separation is deducible from the Axiom of Replacement.

2. Show that the Pair-Set Axiom is deducible from the Axioms of Empty-Set, Power-Set and Replacement.

3. Write down sentences (in the language of ZF) to express the assertions that, for any two sets x and y, the product $x \times y$ and the set of all functions from x to y exist. Indicate how to deduce these sentences from the axioms of ZF.

4. Is it true that if x is a transitive set then the relation $\in$ on x is a transitive relation? Does the converse hold?

5. Let F be a function-class that is an automorphism of $(V, \in)$. Show that F must be the identity.

6. What is the rank of $\{2, 3, 6\}$? What is the rank of $\{\{2, 3\}, \{6\}\}$? Work out the ranks of $\mathbb{Z}$, $\mathbb{Q}$ and $\mathbb{R}$, using your favourite constructions of these objects from ω.

7. Without looking at your notes: does the Axiom of Replacement, as an axiom-scheme, say for each formula p that 'if p is a function-class then the image of any set under p is a set', or does it instead say for each formula p which is a function-class that 'the image of any set under p is a set', or does it not matter because these two are obviously equivalent?

8. A set x is called *hereditarily finite* if each member of $TC(\{x\})$ is finite. Prove that the class HF of hereditarily finite sets coincides with V_ω. Which of the axioms of ZF are satisfied in the structure HF (i.e. the set HF, with the relation $\in |HF$)?

9. Which of the axioms of ZF are satisfied in the structure $V_{\omega+\omega}$?

10. What is the cardinality of the set of all continuous functions from $\mathbb{R}$ to $\mathbb{R}$?

11. Is there an ordinal α such that $\omega_\alpha = \alpha$?

12. Explain why, for each $n \in \omega$, there is no surjection from $\aleph_n$ to $\aleph_{n+1}$. Use this fact to show that there is no surjection from $\aleph_\omega$ to ${\aleph_\omega}^{\aleph_0}$, and deduce that $2^{\aleph_0} \neq \aleph_\omega$.

13. If ZF is consistent then, by Downward Löwenheim-Skolem, it has a countable model. Doesn't this contradict the fact that, for example, the power-set of ω is uncountable?

14. Prove (in ZF) that a countable union of countable sets cannot have cardinality $\aleph_2$.

15. The function-classes $x + y = x \triangle y$ and $xy = x \cap y$ 'make V into a ring (without 1)', in the sense that all of the axioms for a ring hold in this structure. Is it possible to make V into a field?

$^+$16. Show that the function $f(n) = 2^n$ is definable in the language of PA – in other words, find a formula $p(x, y)$ in the language of PA such that, in the natural numbers, $p(n, m)$ holds if and only if $m = 2^n$.