1. Show that $\{\mathbf{Inf}, \mathbf{Sep}\} \vdash \mathbf{Emp}$. Does $\mathbf{Inf} \vdash \mathbf{Emp}$? Does $\mathbf{Sep} \vdash \mathbf{Emp}$? *[N.B. You should interpret* $\{\mathbf{Inf}, \mathbf{Sep}\}$ *as "the collection of formulae in the language of sets consisting of the Axiom of Infinity and every instance of the Axiom of Separation" etc.]*

2. Show that $\mathbf{Rep} \vdash \mathbf{Sep}$. Show also that $\{\mathbf{Emp}, \mathbf{Pow}, \mathbf{Rep}\} \vdash \mathbf{Pair}$.

3. Write down sentences in the language of sets to express the assertions that, for any two sets x and y, the product $x \times y$ and the set of all functions from x to y exist. Indicate how to deduce these sentences from the axioms of ZF.

4. Write down a formula p in the language of sets with $\mathrm{FV}(p) = \{x\}$ that says "x is a (von Neumann) ordinal". What should the von Neumann ordinal ω^2 be? Why do the axioms of ZF prove that it is a set?

5. Is it true that if x is a transitive set then the relation $\in$ on x is a transitive relation? Does the converse hold?

6. What precisely do we mean by the assertion "every set is a class"? Show carefully that if V is a model of ZF then the class V is not a set.

7. Formulate a precise definition of a class defined by a formula p in the language of sets with $FV(p) = \{x, t\}$ where t is a parameter.

8. Let $\in$-**Ind** denote the principle of $\in$-induction and let p be the formula $(\forall y)((x \in y) \Rightarrow (\exists z)((z \in y) \wedge (z \cap y = \emptyset)))$. Show that $(\mathrm{ZF} \setminus \{\mathbf{Fdn}\}) \cup \{\in\text{–}\mathbf{Ind}\} \vdash (\forall x)p$ and hence that $(\mathrm{ZF} \setminus \{\mathbf{Fdn}\}) \cup \{\in\text{–}\mathbf{Ind}\} \vdash \mathbf{Fdn}$.

9. What is the rank of $\{2, 3, 6\}$? What is the rank of $\{\{2, 3\}, \{6\}\}$? Work out the ranks of $\mathbb{Z}$, $\mathbb{Q}$ and $\mathbb{R}$, using your favourite constructions of these objects from ω.

10. A set x is called *hereditarily finite* if each member of $\mathrm{TC}(\{x\})$ is finite. Prove that the class HF of hereditarily finite sets coincides with V_ω. Which of the axioms of ZF are satisfied in the structure HF?

11. Which of the axioms of ZF are satisfied in the structure $V_{\omega+\omega}$?

12. If ZF is consistent then, by Downward Löwenheim-Skolem, it has a countable model. Doesn't this contradict the fact that, for example, the power-set of ω is uncountable?

13. Show that if ZF is consistent then it has a model V containing elements x_0, x_1, x_2, ..., such that $x_{n+1} \in x_n$ for $n = 0, 1, 2, \ldots$. Doesn't this show that ZF is in fact inconsistent, as now $\{x_0, x_1, x_2, \ldots\}$ is a non-empty set by **Rep** with no $\in$-minimal member, contradicting **Fdn**?

14. Is $\mathrm{PA} \cup \{\neg\mathrm{Con(PA)}\}$ ω-consistent?

$^+$15. Show (without assuming the fact that every computable/recursive function is PA-definable) that the function $f\colon \mathbb{N} \to \mathbb{N}$ given by $f(n) = 2^n$ is PA-definable.