1. For which n and m is the complete bipartite graph $K_{n,m}$ Hamiltonian? Is the Petersen graph Hamiltonian?

2. Let G be a graph of order n with $e(G) > \binom{n}{2} - (n-2)$. Prove that G is Hamiltonian.

3. Let G be a bipartite graph with vertex classes X, Y. Show that if G has a matching from X to Y then there exists $x \in X$ such that every edge incident with x extends to a matching from X to Y.

4. Let G be a connected bipartite graph with vertex classes X, Y. Show that every edge of G extends to a matching from X to Y if and only if $|\Gamma(A)| > |A|$ for every $A \subset X$, $A \neq \emptyset, X$.

5. Let A be a matrix with each entry 0 or 1. Prove that the minimum number of rows and columns containing all the 1s of A equals the the maximum number of 1s that can be found with no two in the same row or column.

6. For $r \leq n$, an $r \times n$ *Latin rectangle* is an $r \times n$ matrix, with each entry from $\{1, \ldots, n\}$, such that no two entries in the same row or column are the same. Prove that every $r \times n$ Latin rectangle may be extended to an $n \times n$ Latin square.

7. Show that we always have $\kappa(G) \leq \lambda(G)$. For any positive integers $k \leq l$, construct a graph G with $\kappa(G) = k$ and $\lambda(G) = l$.

8. For a set $B \subset V(G)$ and a vertex a not in B, an *a-B fan* is a family of $|B|$ paths from a to B, disjoint except at a. Show that a graph G (with $|G| > k$) is k-connected if and only if there is an a-B fan for every $B \subset V(G)$ with $|B| = k$ and every vertex a not in B.

9. Let G be a k-connected graph ($k \geq 2$), and let $x_1, \ldots, x_k$ be vertices of G. Show that there is a cycle in G containing all the x_i.

10. For each $r \geq 3$, construct a graph G such that G does not contain K_r but G is not $(r-1)$-partite.

11. A *deleted* K_r consists of a K_r from which an edge has been removed. Show that if G is a graph of order n ($n \geq r+1$) with $e(G) > e(T_{r-1}(n))$ then G contains a deleted K_{r+1}.

12. Let $x_1, \ldots, x_n$ be points in the plane such that no two of them are more than distance 1 apart. Prove that, of the $\binom{n}{2}$ possible pairs of points, at most $n^2/3$ are at distance greater than $1/\sqrt{2}$.

$^+$13. Let G be a (possibly infinite) bipartite graph, with vertex classes X, Y, such that $|\Gamma(A)| \geq |A|$ for every $A \subset X$. Give an example to show that G need not contain a matching from X to Y. Show however that if G is countable and $d(x) < \infty$ for every $x \in X$ then G does contain a matching from X to Y. Does this remain true if G is uncountable?

$^+$14. Let G be an r-regular graph on $2r+1$ vertices. Prove that G is Hamiltonian.