1. Construct a 3-regular graph on 8 vertices. Is there a 3-regular graph on 9 vertices?

2. How many spanning trees does K_4 have?

3. Prove that every connected graph has a vertex that is not a cutvertex.

4. Let G be a graph on n vertices, $G \neq K_n$. Show that G is a tree if and only if the addition of any edge to G produces exactly 1 new cycle.

5. Let $n \geq 2$, and let $d_1 \leq d_2 \ldots \leq d_n$ be a sequence of integers. Show that there is a tree with degree sequence $d_1, \ldots, d_n$ if and only if $d_1 \geq 1$ and $\sum d_i = 2n - 2$.

6. Let $T_1, \ldots, T_k$ be subtrees of a tree T, any two of which have at least one vertex in common. Prove that there is a vertex in all the T_i.

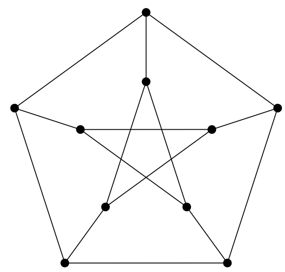
7. Show that every graph of average degree d contains a subgraph of minimum degree at least $d/2$.

8. The *clique number* of a graph G is the maximum order of a complete subgraph of G. Show that the possible clique numbers for a regular graph on n vertices are $1, 2, \ldots, \lfloor n/2 \rfloor$ and n.

9. Let G be a graph on vertex set V. Show that there is a partition $X \cup Y$ of V such that in each of $G[X]$ and $G[Y]$ all vertices have even degree.

10. For which n and m is the complete bipartite graph $K_{n,m}$ planar? When it is not planar, what is its largest (most edges) planar subgraph?

11. Prove that the Petersen graph (shown) is not planar.



12. The *square* of a graph G has vertex set that of G and edge set $\{xy : d(x,y) \leq 2\}$. For which n is the square of the n-cycle planar?

13. Prove that every planar graph has a drawing in the plane in which every edge is a straight-line segment.

$^{+}$14. Each of n elderly dons knows a piece of gossip not known to any of the others. They communicate by telephone, and in each call the two dons concerned reveal to each other all the information they know so far. What is the smallest number of calls that can be made in such a way that, at the end, all the dons know all the gossip?