Mich. 2015 **GRAPH THEORY – EXAMPLES 1** IBL

1. Construct a 3-regular graph on 8 vertices. Is there a 3-regular graph on 9 vertices?

2. How many spanning trees does K_4 have?

3. Prove that every connected graph has a vertex that is not a cutvertex.

4. Let G be a graph on n vertices, $G \neq K_n$. Show that G is a tree if and only if the addition of any edge to G produces exactly 1 new cycle.

5. Let $n \geq 2$, and let $d_1 \leq d_2 \ldots \leq d_n$ be a sequence of integers. Show that there is a tree with degree sequence $d_1, \ldots, d_n$ if and only if $d_1 \geq 1$ and $\sum d_i = 2n - 2$.

6. Let $T_1, \ldots, T_k$ be subtrees of a tree T, any two of which have at least one vertex in common. Prove that there is a vertex in all the T_i.

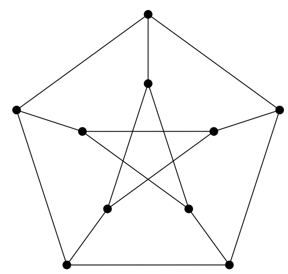
7. Show that every graph of average degree d contains a subgraph of minimum degree at least $d/2$.

8. The *clique number* of a graph G is the maximum order of a complete subgraph of G. Show that the possible clique numbers for a regular graph on n vertices are $1, 2, \ldots, \lfloor n/2 \rfloor$ and n.

9. Let G be a graph on vertex set V. Show that there is a partition $X \cup Y$ of V such that in each of $G[X]$ and $G[Y]$ all vertices have even degree.

10. For which n and m is the complete bipartite graph $K_{n,m}$ planar?

11. Prove that the Petersen graph (shown) is not planar.



12. The *square* of a graph G has vertex set that of G and edge set $\{xy : d(x,y) \leq 2\}$. For which n is the square of the n-cycle planar?

13. Prove that every planar graph has a drawing in the plane in which every edge is a straight-line segment.

+14. Among a group of n dons, any two have exactly one mutual friend. Show that some don is friends with all the others.