1. What is $\mathrm{ex}\,(P)$, where P is the Petersen graph (shown)?

2. For each $r \geqslant 3$, construct a graph of chromatic number r that contains no K_r.

3. Without assuming Turán's theorem, show that the Turán graph $T_r(n)$ is the unique r-partite graph of order n with the largest possible number of edges.

4. Let n, k be positive integers with $k < n/2$. Show that a connected graph G with $|G| = n$ and $\delta(G) \geqslant k$ must contain a path of length $2k$.

5. Show that $\mathrm{ex}\,(n; K_{2,t}) \leqslant \frac{n}{4}\left(1 + \sqrt{1 + 4(n-1)(t-1)}\right)$.

6. What is $\mathrm{ex}\,(n; K_{1,t})$?

7. Let G be a graph of order $n \geqslant 5$ with $e(G) \geqslant \lfloor \frac{n^2}{4} \rfloor + 2$. Show that G must contain two triangles with precisely one vertex in common.

8. Let $x_1, x_2, \ldots, x_{3n}$ be points in the plane such that no two of them are more than distance 1 apart. Prove that at most $3n^2$ of the distances $\|x_i - x_j\|$ $(i < j)$ are greater than $1/\sqrt{2}$.

9. Show that for all $t \geqslant 2$, $z(n, t) = \Theta(\mathrm{ex}\,(n; K_{t,t}))$.

10. Without assuming the Erdős-Stone theorem, show that $\mathrm{ud}\,(G)$ is well-defined for all infinite 'graphs' G.

11. For each positive integer n, let g_n be the largest integer k such that it is possible to colour k edges of the complete graph K_n blue or yellow without creating a monochromatic (blue or yellow) triangle. Show that $g_n/\binom{n}{2}$ converges, and find $\lim_{n \to \infty} g_n/\binom{n}{2}$.

12. How many edges can a graph G of order n containing precisely one triangle have?

13. Let $x_1, x_2, \ldots \in \mathbb{R}^d$ be distinct and, for each n, let $f(n)$ be the number of pairs of points from $x_1, \ldots, x_n$ that are at distance 1: $f(n) = |\{(i,j) : 1 \leqslant i < j \leqslant n, \|x_i - x_j\| = 1\}|$. Show that if $d = 3$ then $f(n)/\binom{n}{2} \to 0$ as $n \to \infty$. Show more generally that if $d \geqslant 3$ then $f(n) \leqslant (1 - \frac{1}{D} + o(1))\binom{n}{2}$, where $D = \lfloor d/2 \rfloor$.

[+]14. Construct a triangle-free graph of chromatic number 1526.