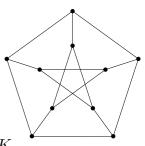
Mich. 2014 GRAPH THEORY—EXAMPLES 2 PAR



- 1. What is ex(P), where P is the Petersen graph (shown)?
- 2. For each $r \ge 3$, construct a graph of chromatic number r that contains no K_r .
- 3. Without assuming Turán's theorem, show that the Turán graph $T_r(n)$ is the unique r-partite graph of order n with the largest possible number of edges.
- 4. Let n, k be positive integers with k < n/2. Show that a connected graph G with |G| = n and $\delta(G) \ge k$ must contain a path of length 2k.
- 5. Show that $ex(n; K_{2,t}) \leq \frac{n}{4} \left(1 + \sqrt{1 + 4(n-1)(t-1)} \right)$.
- 6. What is $ex(n; K_{1,t})$?
- 7. Let G be a graph of order $n \ge 5$ with $e(G) \ge \lfloor \frac{n^2}{4} \rfloor + 2$. Show that G must contain two triangles with precisely one vertex in common.
- 8. Let x_1, x_2, \ldots, x_{3n} be points in the plane such that no two of them are more than distance 1 apart. Prove that at most $3n^2$ of the distances $||x_i x_j||$ (i < j) are greater than $1/\sqrt{2}$.
- 9. Show that for all $t \ge 2$, $z(n,t) = \Theta(\operatorname{ex}(n; K_{t,t}))$.
- 10. Without assuming the Erdős-Stone theorem, show that ud(G) is well-defined for all infinite 'graphs' G.
- 11. For each positive integer n, let g_n be the largest integer k such that it is possible to colour k edges of the complete graph K_n blue or yellow without creating a monochromatic (blue or yellow) triangle. Show that $g_n/\binom{n}{2}$ converges, and find $\lim_{n\to\infty} g_n/\binom{n}{2}$.
- 12. How many edges can a graph G of order n containing precisely one triangle have?
- 13. Let $x_1, x_2, \ldots \in \mathbb{R}^d$ be distinct and, for each n, let f(n) be the number of pairs of points from x_1, \ldots, x_n that are at distance 1: $f(n) = |\{(i,j) : 1 \le i < j \le n, ||x_i x_j|| = 1\}|$. Show that if d = 3 then $f(n)/\binom{n}{2} \to 0$ as $n \to \infty$. Show more generally that if $d \ge 3$ then $f(n) \le (1 \frac{1}{D} + o(1))\binom{n}{2}$, where $D = \lfloor d/2 \rfloor$.
- +14. Construct a triangle-free graph of chromatic number 1526.