1. For which n and m is the complete bipartite graph $K_{n,m}$ Hamiltonian? Is the Petersen graph Hamiltonian?

2. Let G be a graph of order n with $e(G) > \binom{n}{2} - (n-2)$. Prove that G is Hamiltonian.

3. Let G be a bipartite graph with vertex classes X, Y. Show that if G has a matching from X to Y then there exists $x \in X$ such that every edge incident with x extends to a matching from X to Y.

4. Let G be a connected bipartite graph with vertex classes X, Y. Show that every edge of G extends to a matching from X to Y if and only if $|\Gamma(A)| > |A|$ for every $A \subset X$, $A \neq \emptyset, X$.

5. Let A be a matrix with each entry 0 or 1. Prove that the minimum number of rows and columns containing all the 1s of A equals the the maximum number of 1s that can be found with no two in the same row or column.

6. An $n \times n$ *Latin square* (resp. $r \times n$ *Latin rectangle*) is an $n \times n$ (resp. $r \times n$) matrix, with each entry from $\{1, \ldots, n\}$, such that no two entries in the same row or column are the same. Prove that every $r \times n$ Latin rectangle may be extended to an $n \times n$ Latin square.

$^+$7. Let G be a (possibly infinite) bipartite graph, with vertex classes X, Y, such that $|\Gamma(A)| \geq |A|$ for every $A \subset X$. Give an example to show that G need not contain a matching from X to Y. Show however that if G is countable and $d(x) < \infty$ for every $x \in X$ then G does contain a matching from X to Y. Does this remain true if G is uncountable?

8. Show that we always have $\kappa(G) \leq \lambda(G)$. For any positive integers $k \leq l$, construct a graph G with $\kappa(G) = k$ and $\lambda(G) = l$.

9. For a set $B \subset V(G)$ and a vertex a not in B, an *a-B fan* is a family of $|B|$ paths from a to B, any two meeting only at a. Show that a graph G (with $|G| > k$) is k-connected if and only if there is an a-B fan for every $B \subset V(G)$ with $|B| = k$ and every vertex a not in B.

10. Let G be a k-connected graph ($k \geq 2$), and let $x_1, \ldots, x_k$ be vertices of G. Show that there is a cycle in G containing all the x_i.

11. For each $r \geq 3$, construct a graph G such that G does not contain K_r but G is not $(r-1)$-partite.

12. Let G be a graph of order n that does not contain an even cycle. Prove that each vertex x of G with $d(x) \geq 3$ is a cutvertex, and deduce that G has at most $\lfloor 3(n-1)/2 \rfloor$ edges. Give (for each n) a graph for which equality holds. How does this bound compare with the maximum number of edges of a graph of order n containing no *odd* cycles?

13. A *deleted* K_r consists of a K_r from which an edge has been removed. Show that if G is a graph of order n ($n \geq r+1$) with $e(G) > e(T_{r-1}(n))$ then G contains a deleted K_{r+1}.

14. A *bowtie* consists of two triangles meeting in one vertex. Show that if G is a graph of order n ($n \geq 5$) with $e(G) > \lfloor n^2/4 \rfloor + 1$ then G contains a bowtie.

$^+$15. Let G be an r-regular graph on $2r+1$ vertices. Prove that G is Hamiltonian.