Some common discrete distributions

Distribution	Notation	pmf $f(x)$	Range of X	Parameter range	$\mathbb{E}(X)$	$\mathrm{Var}(X)$	$\mathbb{E}(z^X)$
Discrete uniform	$X \sim U\{1,\dots,n\}$	$\frac{1}{n}$	$\{1,\dots,n\}$	$n \in \mathbb{N}$	$\frac{1}{2}(n+1)$	$\frac{1}{12}(n^2-1)$	$\frac{1}{n}\sum_{i=1}^n z^i$
Binomial	$X \sim \mathrm{Bin}(n,p)$	$\binom{n}{x}p^x(1-p)^{n-x}$	$\{0,1,\dots,n\}$	$n \in \mathbb{N}$, $p \in [0,1]$	np	$np(1-p)$	$\{pz+(1-p)\}^n$
Poisson	$X \sim \mathrm{Poisson}(\lambda)$	$e^{-\lambda}\frac{\lambda^x}{x!}$	$\{0,1,\dots\}$	$\lambda \in [0,\infty)$	λ	λ	$e^{\lambda(z-1)}$
Negative binomial	$X \sim \mathrm{NegBin}(k,p)$	$\binom{x-1}{k-1}p^k(1-p)^{x-k}$	$\{k,k+1,\dots\}$	$k \in \mathbb{N}, p \in [0,1]$	$\frac{k}{p}$	$\frac{k(1-p)}{p^2}$	$\frac{(pz)^k}{\{1-(1-p)z\}^k}$
Multinomial	$X \sim \mathrm{Multi}(n,p_1,\dots,p_k)$	$\frac{n!}{n_1!\dots n_k!}p_1^{n_1}\dots p_k^{n_k}$	$(n_1,\dots,n_k) \in$ $\{0,1,\dots,n\}^k : \sum_i n_i = n$	$p_1,\dots,p_k \in [0,1]$: $\sum_i p_i = 1$, $n \in \mathbb{N}$	$(np_1,\dots,np_k)$	$\mathrm{Cov}(X_i,X_j) = \begin{cases} np_i(1-p_i) & i=j \\ -np_ip_j & i\neq j \end{cases}$	$\mathbb{E}(z_1^{X_1}\dots z_k^{X_k}) = (\sum_{i=1}^k p_iz_i)^n$

Notes:

1. The $\mathrm{Bin}(1,p)$ distribution is also called the $\mathrm{Bernoulli}(p)$ distribution. If $X_1,\dots,X_n \overset{iid}{\sim} \mathrm{Bernoulli}(p)$, then $\sum_{i=1}^n X_i \sim \mathrm{Bin}(n,p)$. The $\mathrm{Bin}(n,p)$ distribution models the number of successes in n independent trials, each with probability p of success.

2. The $\mathrm{NegBin}(1,p)$ distribution is also called the $\mathrm{Geometric}(p)$ distribution. If $X_1,\dots,X_k \overset{iid}{\sim} \mathrm{Geometric}(p)$, then $\sum_{i=1}^k X_i \sim \mathrm{NegBin}(k,p)$. The $\mathrm{NegBin}(k,p)$ distribution models the number of independent trials required to attain k successes, each with probability p of success.

3. The $\mathrm{Multi}(n,p_1,\dots,p_k)$ distribution models the number of balls that appear in each of k buckets, when n balls are placed independently in the buckets and a ball falls in the ith bucket with probability p_i.

Some common (absolutely) continuous distributions

Distribution	Notation	pdf $f(x)$	Range	Parameter range	$\mathbb{E}(X)$	$\mathrm{Var}(X)$	$\mathbb{E}(e^{tX})$
Uniform	$X \sim U[a,b]$	$\frac{1}{b-a}$	$[a,b]$	$(a,b) \in \mathbb{R}^2, a < b$	$\frac{1}{2}(a+b)$	$\frac{1}{12}(b-a)^2$	$\frac{e^{bt}-e^{at}}{t(b-a)}$
Normal	$X \sim N(\mu, \sigma^2)$	$\frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{1}{2\sigma^2}(x-\mu)^2}$	$\mathbb{R}$	$\mu \in \mathbb{R}$, $\sigma \in (0,\infty)$	μ	σ^2	$e^{t\mu+\sigma^2 t^2/2}$
Gamma	$X \sim \mathrm{Gamma}(\alpha, \lambda)$	$\frac{\lambda^\alpha x^{\alpha-1} e^{-\lambda x}}{\Gamma(\alpha)}$	$(0,\infty)$	$\alpha \in (0,\infty)$, $\lambda \in (0,\infty)$	$\frac{\alpha}{\lambda}$	$\frac{\alpha}{\lambda^2}$	$\begin{cases} (\frac{\lambda}{\lambda-t})^\alpha & \text{if } t < \lambda \\ \infty & \text{if } t \geq \lambda \end{cases}$
Beta	$X \sim \mathrm{Beta}(a,b)$	$\frac{\Gamma(a+b)}{\Gamma(a)\Gamma(b)} x^{a-1}(1-x)^{b-1}$	$(0,1)$	$a \in (0,\infty)$, $b \in (0,\infty)$	$\frac{a}{a+b}$	$\frac{ab}{(a+b)^2(a+b+1)}$	
Cauchy	$X \sim \mathrm{Cauchy}$	$\frac{1}{\pi(1+x^2)}$	$\mathbb{R}$		Does not exist	∞	$\begin{cases} 1 & \text{if } t = 0 \\ \infty & \text{if } t \neq 0 \end{cases}$
Multivariate normal	$X \sim N_d(\mu, \Sigma)$	$\frac{e^{-\frac{1}{2}(x-\mu)^T \Sigma^{-1}(x-\mu)}}{(2\pi)^{d/2}(\det\Sigma)^{1/2}}$	$\mathbb{R}^d$	$\mu \in \mathbb{R}^d$, Σ pos. def.	μ	$\mathrm{Cov}(X_i, X_j) = \Sigma_{ij}$	$\mathbb{E}(e^{t^T X}) = e^{t^T\mu + t^T\Sigma t/2}$

Notes:

1. The Gamma$(1,\lambda)$ distribution is the same as the Exponential(λ) distribution. If $X_1, \ldots, X_n \overset{iid}{\sim}$ Exponential(λ), then $\sum_{i=1}^n X_i \sim \mathrm{Gamma}(n,\lambda)$.
2. For $n \in \mathbb{N}$, the Gamma$(\frac{n}{2}, \frac{1}{2})$ distribution is the same as the χ^2_n distribution. If $X_1, \ldots, X_n \overset{iid}{\sim} N(0,1)$, then $\sum_{i=1}^n X_i^2 \sim \chi^2_n$.
   If $Y \sim \mathrm{Gamma}(n,\lambda)$ then $2\lambda Y \sim \chi^2_{2n}$.
3. Recall that the Gamma function is defined, for $z \in \mathbb{C}$ with $\mathrm{Re}(z) > 0$, by $\Gamma(z) = \int_0^\infty t^{z-1} e^{-t}\,dt$. If $n \in \mathbb{N}$, then $\Gamma(n) = (n-1)!$. The function $B(a,b) = \frac{\Gamma(a)\Gamma(b)}{\Gamma(a+b)}$ is called the beta function.
4. More generally, we can define the degenerate normal distribution: say $X \sim N(\mu, 0)$ if $\mathbb{P}(X = \mu) = 1$. Then we say $X = (X_1, \ldots, X_d) \sim N_d(\mu, \Sigma)$ if every linear combination $t_1 X_1 + \ldots + t_d X_d$ has a (possibly degenerate) univariate normal distribution. This more general definition includes situations like the following: suppose $X_1 \sim N(0,1)$, and let $X = (X_1, X_1)$. Then $X \sim N_2(0, \Sigma)$, where $\Sigma = \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}$. Note here that $\det\Sigma = 0$.