ANALYSIS AND TOPOLOGY — 2024/2025 SYLLABUS

Sheet 1

A. Analysis on Metric Spaces .... [12 lectures]

1. **Basics**: definition and examples; open sets and metric topology (including topological equivalence of metrics); closed sets and closure; neighborhoods.
2. **Convergent sequences**: definition and examples; pointwise vs. uniform convergence; closure vs. sequential closure.

Examples: the normed vector spaces $(\mathbb{R}^n, \ell_p)$ and $(C([0,1]), L^p)$, the discrete metric.

Sheet 2

3. **Continuous functions**: definition and examples; sequential continuity vs. continuity; homeomorphisms and isometries; continuity vs. uniform continuity; Lipschitz continuity; uniform and Lipschitz equivalence of metrics.
4. **Completeness**: definition and examples; contraction mappings; some applications of contraction mapping principle, including Picard–Lindelöf theorem for ODEs.
5. **Compactness**: definition and examples; sequential compactness vs. compactness; Heine–Borel theorem; continuous functions on compact sets; (in sheets) some applications.

Sheet 3

B. Analysis on Topological Spaces .... [7 lectures]

1. **Basics**: definitions and examples; base of a topology; nice topologies (Hausdorff property, second countability, metrizability); closed sets; neighborhoods; accumulation points and closure.
2. **Convergent sequences**: definition and examples; sequential closure vs. closure.
3. **Continuous functions**: definition and examples; sequential continuity vs. continuity; homeomorphisms and topological invariants.

Examples: the discrete topology, the indiscrete topology.

4. **Three inherited topologies**: subspace topology; product topology; quotient topology.
5. **Compactness**: definition and examples; sequential compactness vs. compactness; continuous functions on compact sets; compactness under products and quotients.

Sheet 4

6. **Connectedness**: definitions and examples; continuous functions on connected spaces; connectedness under products and quotients; path-connectedness; some applications in the context of *metric spaces*.

C. Analysis on $\mathbb{R}^n$ .... [5 lectures]

1. **Differentiation**: first derivative; inverse (+ statement of implicit) function theorem; differentiation meets uniform convergence; higher order derivatives; some applications, including to functions over matrix spaces.
2. **Power series and series of functions** ($n = 1$): pointwise and absolute convergence and uniform convergence; term-by-term differentiation and integration.

Note: this document $\subseteq$ the official schedules, but the ordering here reflects the order of lectures and examples sheets.

Email comments and suggestions to `rbdt2@cam.ac.uk`