1. The numbers 3,5,7 are all prime; does it ever happen again that three numbers of the form $n, n+2, n+4$ are all prime?

2. Between 10 and 20 there are 4 primes; does it ever happen again that there are 4 primes between two consecutive multiples of 10 (apart from between 0 and 10)?

3. Consider the sequence $41, 43, 47, 53, 61, \ldots$ (where each difference is 2 more than the previous one). Are all of these numbers prime?

4. Does there exist a block of 100 consecutive positive integers, none of which is prime?

5. Show that $2^{19} + 5^{40}$ is not prime. Show also that $2^{91} - 1$ is not prime.

6. If n^2 is a multiple of 3, must n be a multiple of 3?

7. Show that, for every positive integer n, the number $3^{3n+4} + 7^{2n+1}$ is a multiple of 11.

8. Write down carefully (while not looking at your notes) a proof that there are infinitely many primes. By considering numbers of the form $4p_1p_2 \ldots p_k - 1$, prove that there are infinitely many primes of the form $4n-1$. What would go wrong if we tried a similar proof to show that there are infinitely many primes of the form $4n+1$?

9. Prove that $2^{2^n} - 1$ has at least n distinct prime factors.

10. We are given an operation $*$ on the positive integers, satisfying
(i) $1 * n = n + 1$ for all n
(ii) $m * 1 = (m-1) * 2$ for all $m > 1$
(iii) $m * n = (m-1) * (m * (n-1))$ for all $m, n > 1$.
Find the value of $5 * 5$.

11. Suppose that we have some positive integers (not necessarily distinct) whose sum is 100. How large can their product be?

12. Find a positive integer a such that, for every positive integer n, the number $n^4 + a$ is not prime.

13. The *repeat* of a positive integer is obtained by writing it twice in a row (so for example the repeat of 254 is 254254). Is there a positive integer whose repeat is a square number?

14. Some red sweets and blue sweets are distributed among 99 bags. Gareth wants to select 50 of the bags in such a way that he obtains at least half of the red sweets and at least half of the blue sweets. Is he always able to do this?

$^+$15. Each of n elderly dons knows a piece of information not known to any of the others. They communicate by telephone, and in each call the two dons concerned reveal to each other all the information they know so far. What is the smallest number of calls that can be made in such a way that, at the end, all the dons know all the information?