1. For which $n \in \mathbb{N}$, if any, are three numbers of the form $n, n+2, n+4$ all prime?

2. Between 0 and 10 there are four primes. Another example of two consecutive multiples of ten, between which there are four primes, is 10 and 20. Are there further examples?

3. If n^2 is a multiple of 3, must n be a multiple of 3?

4. Write down the negations of the following statements (where $m, n, a, b \in \mathbb{N}$):

(i) if the last Trump doesn't sound then May will be queen

(ii) $\forall m\, \exists n\, \forall a\, \forall b\ \ (n \geq m)\ \wedge\ [(a = 1) \vee (b = 1) \vee (ab \neq n)]$

Is (ii) true?

5. How large can the product of some (not necessarily distinct) natural numbers be, if their sum is 100?

6. Are all numbers in the sequence 41, 43, 47, 53, 61, 71, 83, 97, . . . prime?

7. Is there a power of 2 beginning with 7?

8. Find the highest common factor of 12345 and 54321. Find $u, v \in \mathbb{Z}$ with $76u + 45v = 1$. Does $3381x + 2646y = 21$ have an integer solution?

9. Find the convergents to the fraction $\frac{152}{90}$. Find all integer solutions of $152x + 90y = 2$.

10. Let $a, b, c, d \in \mathbb{N}$. Must the numbers $(a, b)(c, d)$ and (ac, bd) be equal? If not, must one be a factor of the other? If $(a, b) = (a, c) = 1$, must we have $(a, bc) = 1$?

11. Show that, for any $a,\ b \in \mathbb{N}$, the number $\ell = ab/(a, b)$ is an integer (called the *least common multiple* of a and b). Show also that ℓ is divisible by both a and b, and that if $n \in \mathbb{N}$ is divisible by both a and b then $\ell \mid n$.

[Give two proofs, one based on uniqueness of prime factorization, the other Bezout-like.]

12. The *repeat* of a natural number is obtained by writing it twice in a row (for example, the repeat of 356 is 356356). Is there a number whose repeat is a perfect square?

13. Let $a < b$ be distinct natural numbers. Prove that every block of b consecutive natural numbers contains two distinct numbers whose product is divisible by ab. Suppose now $a < b < c$. Must every block of c consecutive numbers contain three distinct numbers whose product is divisible by abc?

14. Is there a power of 2 that begins with 1867?