

1. Prove that $A \setminus (B \cup C) = (A \setminus B) \cap (A \setminus C)$.
2. The *symmetric difference* $A \Delta B$ of two sets A and B is the set of elements that belong to exactly one of A and B . Express this in terms of \cup , \cap and \setminus . Prove that Δ is associative.
3. Let A_1, A_2, A_3, \dots be sets such that $A_1 \cap A_2 \cap \dots \cap A_n \neq \emptyset$ holds for all n . Must it be that $\bigcap_{n=1}^{\infty} A_n \neq \emptyset$?
4. Prove that $f \circ g$ is injective if f and g are injective. Does $f \circ g$ injective imply f injective? Does it imply g injective? What if we replace ‘injective’ by ‘surjective’ everywhere?
5. Let $A = \{1, 2, 3\}$ and $B = \{1, 2, 3, 4, 5\}$? How many functions $A \rightarrow B$ are there? How many are injections? Count the number of surjections $B \rightarrow A$.
6. Let $f : X \rightarrow Y$. Let $A, B \subset X$ and $C, D \subset Y$. For each of claims (a)–(f) below, give a proof or counter-example.

(a) $f(A \cup B) = f(A) \cup f(B)$	(b) $f^{-1}(C \cup D) = f^{-1}(C) \cup f^{-1}(D)$
(c) $f(A \cap B) = f(A) \cap f(B)$	(d) $f^{-1}(C \cap D) = f^{-1}(C) \cap f^{-1}(D)$
(e) $f^{-1}(f(A)) = A$	(f) $f(f^{-1}(C)) = C$

Show that each false claim can be made true by replacing ‘=’ by either ‘ \subset ’ or ‘ \supset ’.
7. Define a relation R on \mathbb{N} by setting aRb if $a \mid b$ or $b \mid a$. Is R an equivalence relation?
8. The relation S contains the relation R if aSb whenever aRb . Let R be the relation on \mathbb{Z} ‘ aRb if $b = a + 3$ ’. How many equivalence relations on \mathbb{Z} contain R ?
9. Show that any collection of pairwise disjoint discs in the plane is countable. What happens if we replace ‘discs’ by ‘circles’?
10. Show that the collection of all finite subsets of \mathbb{N} is countable. What goes wrong if we try to use the diagonal argument to show that it is uncountable?
11. A function $f : \mathbb{N} \rightarrow \mathbb{N}$ is *increasing* if $f(n + 1) \geq f(n)$ for all n and *decreasing* if $f(n + 1) \leq f(n)$ for all n . Is the set of increasing functions countable or uncountable? What about the set of decreasing functions?
12. Find an injection $\mathbb{R}^2 \rightarrow \mathbb{R}$. Is there an injection from the set of all real sequences to \mathbb{R} ?
13. Find a bijection $f : \mathbb{Q} \rightarrow \mathbb{Q} \setminus \{0\}$. Can f be strictly increasing (that is, $f(x) < f(y)$ whenever $x < y$)?
14. Let $S \subset \mathcal{P}\mathbb{N}$ be such that if $A, B \in S$ then $A \subset B$ or $B \subset A$. Can S be uncountable? Is there an uncountable family $T \subset \mathcal{P}\mathbb{N}$ such that $A \cap B$ is finite for all distinct $A, B \in T$?
15. Each of an infinite sequence of Trappist set theorists is going to a party, where each will receive a coloured hat, either red or blue. Each person will be able to see every hat but his own. After all hats are assigned, each set theorist must write down (in silence, obviously) a guess as to his own hat colour. You are asked to supply them with a strategy such that, should they follow it, only finitely many of them will guess wrongly. Can you do it?