1. Find the highest common factor of 12345 and 54321. Find $u, v \in \mathbb{Z}$ with $76u + 45v = 1$. Does $3381x + 2646y = 21$ have an integer solution?

2. Find the convergents to the fraction $\frac{57}{44}$. Prove that if x and y are integers such that $57x + 44y = 1$, then $x = 17 - 44k$ and $y = 57k - 22$ for some $k \in \mathbb{Z}$.

3. Let $a, b, c \in \mathbb{N}$. Must the numbers $(a, b)(c, d)$ and (ac, bd) be equal? If not, must one be a factor of the other? If $(a, b) = (a, c) = 1$, must we have $(a, bc) = 1$?

4. Show that, for any $a,\ b \in \mathbb{N}$, the number $\ell = ab/(a, b)$ is an integer (called the *least common multiple* of a and b). Show also that ℓ is divisible by both a and b, and that if $n \in \mathbb{N}$ is divisible by both a and b then $\ell \mid n$.

5. Do there exist 100 consecutive natural numbers none of which is prime?

6. In the sequence 41, 43, 47, 53, 61, . . . , each difference is two more than the previous one. Are all the numbers in the sequence prime?

7. Let A be a set of n positive integers. Show that every sequence of 2^n numbers taken from A contains a consecutive block of numbers whose product is a square. (For instance, 2,5,3,2,5,2,3,5 contains the block 5,3,2,5,2,3.)

8. Use the inclusion-exclusion principle to count the number of primes less than 121.

9. How many subsets of $\{1, 2, \ldots, n\}$ are there of even size?

10. By suitably interpreting each side, or otherwise, establish the identities

$$\binom{k}{k} + \binom{k+1}{k} + \binom{k+2}{k} + \cdots + \binom{n}{k} = \binom{n+1}{k+1}$$

$$\binom{n}{0}^2 + \binom{n}{1}^2 + \cdots + \binom{n}{n}^2 = \binom{2n}{n}$$

11. A *triomino* is an L-shaped pattern made from three square tiles. A $2^k \times 2^k$ chessboard, whose squares are the same size as the tiles, has one of its squares painted puce. Show that the chessboard can be covered with triominoes so that only the puce square is exposed.

12. By considering the number of ways to partition a set of order $2n$ into n parts of order 2, show that $(n + 1)(n + 2) \ldots (2n)$ is divisible by 2^n but not by 2^{n+1}.

13. The *repeat* of a natural number is obtained by writing it twice in a row (for example, the repeat of 356 is 356356). Is there a number whose repeat is a perfect square?

14. Let $a < b$ be distinct natural numbers. Prove that every block of b consecutive natural numbers contains two distinct numbers whose product is divisible by ab. Suppose now $a < b < c$. Must every block of c consecutive numbers contain three distinct numbers whose product is divisible by abc?