

1. For which $n \in \mathbb{N}$, if any, are three numbers of the form $n, n + 2, n + 4$ all prime?
2. Between 0 and 10 there are four primes. The same is true between 10 and 20. Are there two other consecutive multiples of ten between which there are four primes?
3. If n^2 is a multiple of 3, must n be a multiple of 3?
4. Write down the negations of the following assertions (where $m, n, a, b \in \mathbb{N}$):
 - (i) if Coke is not worse than Pepsi then Osborne hasn't a clue what he's about.
 - (ii) $\forall m \exists n \forall a \forall b (n \geq m) \wedge [(a = 1) \vee (b = 1) \vee (ab \neq n)]$,
5. The sum of some (not necessarily distinct) natural numbers is 100. How large can their product be?
6. Prove that $A \setminus (B \cup C) = (A \setminus B) \cap (A \setminus C)$.
7. The *symmetric difference* $A \Delta B$ of two sets A and B is the set of elements that belong to exactly one of A and B . Express this in terms of \cup , \cap and \setminus . Prove that Δ is associative.
8. Let A_1, A_2, A_3, \dots be sets such that $A_1 \cap A_2 \cap \dots \cap A_n \neq \emptyset$ holds for all n . Must it be that $\bigcap_{n=1}^{\infty} A_n \neq \emptyset$?
9. Prove that $f \circ g$ is injective if f and g are injective. Does $f \circ g$ injective imply f injective? Does it imply g injective? What if we replace 'injective' by 'surjective' *passim*?
10. Let $A = \{1, 2, 3\}$ and $B = \{1, 2, 3, 4, 5\}$? How many functions $A \rightarrow B$ are there? How many are injections? Count the number of surjections $B \rightarrow A$.
11. Let $f : X \rightarrow Y$ and let $C, D \subset Y$. Prove that $f^{-1}(C \cap D) = f^{-1}(C) \cap f^{-1}(D)$. Let $A, B \subset X$. Must it be true that $f(A \cap B) = f(A) \cap f(B)$?
12. Define a relation R on \mathbb{N} by setting aRb if $a \mid b$ or $b \mid a$. Is R an equivalence relation?
13. The relation S *contains* the relation R if aSb whenever aRb . Let R be the relation on \mathbb{Z} ' aRb if $b = a + 3$ '. How many equivalence relations on \mathbb{Z} contain R ?
14. Find a bijection $f : \mathbb{Q} \rightarrow \mathbb{Q} \setminus \{0\}$. Can f be strictly increasing (that is, $f(x) < f(y)$ whenever $x < y$)?