ERC PROJECT SINGWAVE

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The 6th Clay Millenium has a very simple formulation: does there exist three dimensional incompressible fluid singularities? The SingWave project aims at a better mathematical understanding of singularity formation mechanisms in non linear waves propagation which lie at the heart of an intense international activity.

The initial aim of the project was to extend our understanding of these questions in so called energy super critical regimes for classical models like the non linear Schrödinger equation, which are a natural starting point for the exploration of more fluid models. This last direction of investigation has been a success and has led to the discovery of new singularity formation mechanisms for viscous compressible fluids and defocusing models.

Main achievements of the project

On critical non linear Schrödinger models, we obtain in [7] the first existence result of infinite time blow up solutions which was a long standing open problem, and blow up occurs through a new collision of solitary waves process. We construct in [1] the family of slightly super critical blow up profiles which is a delicate ode problem.

On energy super critical parabolic problems, we developp in [6] a new spectral approach for the study of non self similar blow up, here applied to the melting Stefan problem. This robust methodology is at the heart of [2, 13] which describes a completely new anisotropic blow up mechanism for the non linear heat equation. Classification of the flow near the solitary wave in the energy critical case in large dimension is obtained in [3].

On the weakly turbulent mechanisms, we have obtained in [5] the first result of explicit growth of Sobolev norms growth for the cubic half wave equation, which extends on previous results by Gerard and Grelier on integrable cases. We started the adaptation of this program with Naumkin in [9] and obtained new results on the existence of non local travelling waves which are the starting point of this approach. In [4], we answer a classical question on the existence of decaying in time smooth well localized time dependent potentials which lead to growth of Sobolev norms for the perturbed harmonic oscillator. The underlying mechanism is new and based on a blow up bubble approach (as opposed to Fourier methods).

Compressible fluids. The series of preprints [12, 11, 10] obtained with three close collaborators of the project (Rodnianski, Merle and Szeftel) appeared on archiv in Dec 2019. These works answer two classical problems. First we prove the existence of blow up dynamics for the defocusing energy super critical non linear Schrödinger equations which is a long standing open problem. The blow up mechanism is completely new and highly oscillatory, and relies on the study of an underlying leading order compressible Euler like flow. Hence the link with the second result which is the first description of viscous three dimensional compressible shock waves. It was widely believed that viscosity should have an immense effect on singularity formation, as it does in dimension one and two, we prove the exact opposite: in suitable regimes, three dimensional viscous shocks may be governed by non viscous Euler type shocks. The analysis relies on a delicate study of the flow in a new quasilinear setting and designs a universal route map for the construction of blow up bubbles. These works demonstrate the relevance of the approach developed by the SingWave project for the study of fluid mechanics problems, and quasilinear waves in general.

Collaborators funded by the project

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Yvan Naumkin, post doctorate.

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Activities funded by the project

The project has funded a seminar on PDE's in Laboratoire J.A. Dieudonné (Nice). The project has funded several schools in Europe (Roma, Vienna, La Thuile, St Etienne de Tinée) and has participated in the support of an international multidisciplinary conference on waves in Nice, June 2019.

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