KUWAIT FOUNDATION LECTURE 50 - October 25, 2005

Existence theorems and numerical solutions in complex differential geometry

Simon Donaldson
(Imperial College)

An important theme in Kahler geometry is the search for an optimal metric on a complex projective algebraic variety. In the 1950's, Calabi formulated a series of conjectures dealing with the Ricci tensor and volume form of a Kahler manifold. These were subsequently proved by Yau. The most celebrated case is the existence of 'Calabi-Yau metrics' on varieties with $c_1 = 0$. In the 1980's, Calabi initiated another line of work - the question of the existence of 'extremal' Kahler metrics. The most important examples of these are constant scalar curvature metrics. While there is now a setting in place for understanding this existence question, general existence theorems are difficult, due to the complexity of the PDE involved.

Alongside the existence question one can ask for explicit numerical approximations to these special metrics. There is a general scheme for constructing these using embedding, $X \to \mathbb{CP}^{N_K}$ with $N_K \to \infty$. For each h' one seeks a 'balanced' metric on the underlying $\mathbb{C}^{N_k+1}$. In this lecture some numerical results were presented for the K3 surface X which is the double cover of the plane branched over the curve $x^6 + y^6 + z^6 = 0$. These gave useful approximations to the Calabi-Yau metric depending on 11 and 26 real parameters.

Existence Theorems and numerical solutions in complex differential geometry.

An important theme in Kahler geometry is the search for an optimal metric on a complex projective algebraic variety. In the 1950's, Calabi formulated a series of conjectures dealing with the Ricci tensor and volume form of a Kahler manifold. These were subsequently proved by Yau. The most celebrated case is the existence of "Calabi-Yau metrics" on varieties with $c_1 = 0$. In the 1980's, Calabi initiated another line of work — the question of the existence of "extremal" Kahler metrics. The most important examples of these are constant scalar curvature metrics. While there is now a setting in place for understanding this existence question, general existence theorems are difficult, due to the complexity of the PDE involved.

Alongside the existence question one can ask for explicit numerical approximations to these special metrics. There is a general scheme for constructing these using embeddings $X \to \mathbb{CP}^{N_k}$ with $N_k \to \infty$. For each k one seeks a "balanced" metric on the underlying $\mathbb{C}^{N_k+1}$. In the lecture some numerical results were presented for the K3 surface X which is the double cover of the plane branched over the curve $x^6+y^6+z^6=0$. These gave useful approximations to the Calabi-Yau metric depending on 11 and 26 real parameters.

Simon Donaldson