KUWAIT FOUNDATION LECTURE 8 - May 23, 2000

THE ADAMS SPECTRAL SEQUENCE FOR ALGEBRAIC HOMOTOPY GROUPS OF SPHERES AND THE MILNOR CONJECTURE ON INVARIANTS OF QUADRATIC FORMS

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Let $n \in \mathbb{Z}$. Recall that the n-th stable stem is defined to be the inductive limit

$$\pi_n^S := \varinjlim_{r \gg 0} \pi_{n+r}(S^r).$$

One has $\pi_n^S = 0$ for $n < 0$ and $\pi_0^S = \mathbb{Z}$, this isomorphism being induced by the degree isomorphisms: $\pi_n(S^n) \xrightarrow{\cong} \mathbb{Z}$, $n \geqslant 1$.

Let k be a field. According to V. Voevodsky and myself there is a reasonable homotopy theory of algebraic varieties over k in which the affine line $\mathbb{A}^1$ plays the role of the unit interval [0,1] in topology. Let [,] denote "homotopy classes" in this homotopy theory of algebraic varieties.

Set $\pi_0^S(k) := \varinjlim_n [(\mathbb{P}^1)^{\wedge n}, (\mathbb{P}^1)^{\wedge n}]$.

Let $GW(k)$ denote the Grothendieck-Witt ring of isomorphism classes of quadratic forms over k. There is a unique ring homomorphism (conjectured to be an isomorphism)

$$GW(k) \to \pi_0^S(k)$$

which maps the quadratic form of rank one $\langle u \rangle = uX^2, u \in K$ to the homotopy class of $f_u : \mathbb{P}^1 \to \mathbb{P}^1, (x, y) \mapsto [ux, y]$. The analogue of J.F. Adams' sequence based on Suslin-Voevodsky mod 2 motivic cohomology then defines, because it degenerates in the critical area, invariants $I(k)^s/I(k)^{s+1} \to K_s^M(k)/2$, reproving Milnor's conjecture.

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The Adams spectral sequence for algebraic homotopy groups of spheres and the Milnor conjecture on invariants of quadratic forms.

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Fabien Morel, May 23rd, 2000