KUWAIT FOUNDATION LECTURE 79 - May 20, 2008

Geometric Analysis on Singular Spaces

Rafe Mazzeo
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The development of new tools in linear PDEs, particularly those from geometric microlocal analysis, is now making it possible to attack a number of appealing problems in geometric analysis – much studied in the smooth compact setting, formulated as problems on various classes of singular spaces. I am only interested in spaces with rather mild and well-structured singularities rather than those satisfying only weak hypotheses on distances, volume properties, etc.

Rather than giving a broad survey of the field and methods, I presented three case studies:

A) An explanation of a well-known anomaly for the scale-invariant t^0 coefficient in the McKean-Singer asymptotics for the heat kernel of the Laplacian on a triangle (or any piecewise C^∞ domain). The explanation involves a smoothing of the domain, a parabolic blow-up in the time-parameter space, and an interpolating function which is a renormalized heat trace on a certain noncompact domain.

B) The moduli space of convex polyhedra in $\mathbb{H}^3$ is still not well-understood. J. J. Stoker conjectured in 1967 that a good set of parameters lies entirely within the set of dihedral angles between faces. I discussed a recent joint result with Montcouquiol settling a local version. The key idea is to treat the polyhedron as a singular Einstein space, and to view this as an extension of the classical Weil-Calabi rigidity theorem.

C) It has long been understood how to flow compact embedded plane curves by their curvature. Only recently has it become possible to describe the flow of networks of curves. The key new idea is to describe the behaviour of high-value vertices and how they instantaneously break apart into a tree which then flows in a simpler way.

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