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On the proof of Serre's conjecture

J. P. Wintenberger
(Université Louis Pasteur, Strasbourg)

The lecture is based on the proof of Serre's conjecture obtained in colloboration with C. Khare. Serre's conjecture states that a representation

$$\bar{\rho} : G_Q \to GL_2(\bar{F}_p)$$

which is irreducible and odd arises from a modular form. Here G_Q is the Galois group of the field of national numbers Q, and $\bar{F}_p$ is the algebraic closure of the field F_p with p elements. Odd means that the image of complex conjugation has determinant -1. A modular form here is a primitive modular form f in $S_k(\Gamma_1(N))$. To f and an embedding ι of the field of coefficients of f in $\bar{Q}_p$, Deligne associates a Galois representation

$$\rho_{f,\iota} : G_Q \to GL_2(\bar{Q}_p)$$

which is irreducible and odd. The theorem says that given $\bar{\rho}$, there is an f and an ι such that $\bar{\rho}$ is isomorphic to the reduction of $\rho_{f,\iota}$. The proof relies on a generalisation of the theorem of modularity of Wiles: $\bar{\rho}$ modular, a lift ρ of $\bar{\rho}$ which is geometric is modular, and also uses systematically the trick of changing primes of Wiles.

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On the proof of Serre's Conjecture by JP Wintenberger

The lecture is on the proof of Serre's conjecture obtained in collaboration with C. KHARE. Serre's conjecture states that a representation $\bar{\rho} : G_{\mathbb{Q}} \to Gl_2(\bar{\mathbb{F}}_p)$ which is irreducible and odd arises from a modular form. Here $G_{\mathbb{Q}}$ is the Galois group of the field of rational numbers $\mathbb{Q}$, $\bar{\mathbb{F}}_p$ is the algebraic closure of the field $\mathbb{F}_p$ with p elements. Odd means that the image of complex conjugation has determinant -1. A modular form here is a primitive modular form f in $S_k(\Gamma_1(N))$. To f and an embedding ι of the field of coefficients of f in $\bar{\mathbb{Q}}_p$, Deligne associates a Galois representation $\rho_{f,\iota} : G_{\mathbb{Q}} \to Gl_2(\bar{\mathbb{Q}}_p)$ which is irreducible and odd. The theorem says that given $\bar{\rho}$ there is an f and a ι such that $\bar{\rho}$ is isomorphic to the reduction of $\rho_{f,\iota}$.

The proof relies on generalizations of the theorem of modularity of Wiles: $\bar{\rho}$ modular, a lift ρ of $\bar{\rho}$ which is geometric is modular, and, also uses systematically the trick of changing prime of Wiles.