KUWAIT FOUNDATION LECTURE 42 - October 19, 2004

GALOIS REPRESENTATIONS IN ARITHMETIC GEOMETRY

Takeshi Saito

(University of Tokyo)

In arithmetic geometry, Galois representations have been playing prominent roles, e.g. in the proof of Fermat's last theorem. In the talk, I recalled basic properties of the ℓ-adic representation on etale cohomology, mostly in connection with the Hasse-Weil L-functions. In the last part of the lecture, I discussed some open problems at the primes of bad reduction and recent progress on the independence of ℓ.

For a projective smooth variety X over $\mathbb{Q}$ and a prime ℓ, the action of the geometric Frobenius Fr_p on $H^m(X_{\overline{\mathbb{Q}}}, \mathbb{Q}_\ell)$ for a prime $p \neq \ell$ where X has good reduction is described by the congruence zeta function of the reduction of X modulo p as a consequence of the Weil conjecture. Consequently, the Hasse-Weil zeta function $L(H^m(X), s)$ is the L-function of the ℓ-adic representation $H^m(X_{\overline{\mathbb{Q}}}, \mathbb{Q}_\ell)$ except at most finitely many bad factors. The right definition of the bad factors and the constant term in the functional equations were given by Serre in the 1960's. However, it is not yet known if the definition is independent of ℓ, in general. The result stated in the lecture is that the weight-monodromy conjecture together with the algebraicity of the Künneth projectors implies the independence of ℓ. The proof, not given in the lecture, uses an alteration and functorial properties of the weight spectral sequences.

Kuwait Foundation Lecture #42

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Takeshi Saito (Tokyo)

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Takeshi Saito Oct. 21, '04.