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CONTINUOUS RANDOM TREES AND APPLICATIONS

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Continuous random trees arise as scaling limits of discrete trees conditioned to be large. For instance, one may consider a discrete tree chosen uniformly at random among all rooted trees on n labelled vertices, or a critical Galton-Watson branching tree conditioned to have total progeny equal to n. In both cases, scaling edges by the factor $1/\sqrt{n}$ and passing to the limit $n \to \infty$ yield a continuous tree called the CRT (Aldous 1993). Formally, the CRT is a random variable taking values in the space of all rooted compact $\mathbb{R}$-trees (where, roughly speaking, an $\mathbb{R}$-tree is a metric space where any two points are joined by a unique compact arc, which is isometric to a line segment). If $(e(s), 0 \leqslant s \leqslant 1)$ denotes a normalized Brownian excursion, the CRT is the (random) metric space $([0,1]/\sim, d)$ where $d(s,s') = e(s) + e(s') - 2 \inf_{[s,s']} e(r)$ and $s \sim s'$ if and only if $d(s,s') = 0$, and the preceding convergence holds in the sense of the Gromov-Hausdorff distance.

Combining the genealogical structure of the CRT with spatial motion leads to the Brownian snake: To every $s \in [0,1]$ is attached a Gaussian variable W_s in $\mathbb{R}^d$, in such a way that $W_s - W_{s'}$ has covariance matrix $d(s,s')\mathrm{Id}$. The Brownian snake is a powerful tool in the probabilistic study of certain semi-linear PDE. A typical application is a classification theorem for all non-negative solutions of $\triangle u = u^2$ is a smooth domain (Mselati 2003).

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Continuous random trees and applications

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