KUWAIT FOUNDATION LECTURE 3 - January 18, 2000

# THE LOCAL LANGLANDS CONJECTURE


## Richard Taylor
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The local Langlands conjecture predicts that for a $p$-adic field $K$ there is a natural bijection between irreducible smooth representations of $\mathrm{GL}_n(K)$ and $n$-dimensional Frobenius semi-simple WD-representations of the Weil group $W_K$. Note that irreducible smooth representations of $\mathrm{GL}_n(K)$ are usually infinite dimensional, and that WD-representations of $W_K$ are very closely related to representations of $\mathrm{Gal}(\overline{K}/K)$. The case $n = 1$ is nothing but local class field theory.

The main content of this conjecture is in the word *natural*. There are various possible interpretations of this, but the standard one (particularly the matching of $L$ and $\varepsilon$-factors of pairs) was made precise by Henniart, who showed that at most one bijection was natural in his sense. M. Harris and I prove the existence of a bijection which is natural in Henniart's sense and which is also compatible with many instances of the global Langlands correspondence. Our correspondence is however not explicit.

Our method is a generalisation of the Lubin-Tate approach to local class field theory. We consider the cohomology of the $p$-adic analytic space associated to the universal deformation space of the unique height $n$ 1-dimensional formal $\mathcal{O}_K$-module over $\overline{\mathbb{F}}_p$ with Drinfeld level structure. In the limit this cohomology has an action of a group very close to $\mathrm{GL}_n(K) \times D_n^\times \times W_K$, where $D_n$ is the division algebra centre $K$ with Hasse invariant $1/n$. We show that the irreducible constituents of this representation realise both the Jacquet-Langlands correspondence between representations of $\mathrm{GL}_n(K)$ and $D_n^\times$ and the local Langlands correspondence between representations of $\mathrm{GL}_n(K)$ and $W_K$. The case $n = 2$ had been worked out by Deligne previously.

# Kuwait Fund Lecture 3

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# "The local Langlands conjecture"

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Richard Taylor 19/1/00