KUWAIT FOUNDATION LECTURE 27 - December 3, 2003

PERIODS AND LINEAR DIFFERENTIAL EQUATIONS

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The *periods* in this lecture are integrals like

$$\sqrt{\pi} = \int_{-\infty}^{\infty} e^{-t^2} dt; \quad (e^{2\pi i a} - 1)\Gamma(a) = \int_{0 \leftrightarrows \infty} e^{-t} t^a \frac{dt}{t}; \quad \mathrm{Bessel}_n(z) = \int_{|z|=1} e^{\frac{z}{2}(u - \frac{1}{u})} \frac{du}{u^{n+1}}.$$

The programme (joint with H. Esnault and A. Beilinson) is to interpret such integrals in terms of local and global *epsilon factors* associated to linear differential equations (connections) on algebraic curves.

One starts with $(X, S, E, \nabla)/k$, where $k \subset \mathbb{C}$, X is an algebraic curve, $S \subset X$ is a finite set, E is a vector bundle, $\nabla : E \to E \otimes \Omega^1_X(*S)$ is a connection with poles along S. One constructs a perfect pairing $H^1_{\mathrm{DR}}(X - S; E, \nabla) \times H_1^{\mathrm{rd}}(\mathcal{E}^\vee) \to \mathbb{C}$; where H_1^{rd} is an homology group built from chains with values in $\mathcal{E}^\vee =$ horizontal sections of $E^\vee_{\mathrm{an}}$. The "rd" stands for *rapid decay.* We assume given $F \subset \mathbb{C}$, and a suitable (compatible with Stokes structure) reduction of structure $\mathcal{E}^\vee = \mathcal{E}^\vee_F \otimes_F \mathbb{C}$. Then H_1^{rd} has an F-structure. Since H^1_{DR} has a k-structure, the determinant of the above pairing gives an invariant in $\mathbb{C}^\times / k^\times F^\times$.

The goal is to use the Witten-Laumon method of Fourier transform in order to give a formula for this period determinant via local terms depending on the choice of a meromorphic 1-form, precisely as in the classical theory of epsilon factors.

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Spencer BLOCH

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