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The characteristic polynomials of automorphisms of lattices

If V is a non-degenerate orthogonal space of dimension $2n$ over the field k , and $T: V \rightarrow V$ is an orthogonal transformation with $\det T = 1$, the characteristic polynomial $f(x) = \det(xI - T)$ of T is palindromic (or reciprocal), in the sense that $f(x) = x^{2n} f(x^{-1})$. The roots λ and λ^{-1} of $f(x)$ occur with the same multiplicity, and the roots ± 1 fixed by this involution occur with even multiplicity.

If $f(x)$ is separable, and we define the monic polynomial $g(y)$ by the identity $f(x) = x^n g(x+x^{-1})$, then f determines an extension of étale algebras:

$$\begin{array}{c} K = k[x]/f(x) \\ \uparrow \\ K^+ = k[y]/g(y) \\ \uparrow \\ k \end{array}$$

The extension K/K^+ is Galois, with group generated by the involution taking x to x^{-1} . The discriminant of K/K^+ has norm $f(1)f(-1)$ in k^* .

These considerations play a role in the study of automorphisms of even, unimodular lattices L , where $V = L \otimes \mathbb{Q}$ and the quadratic extension K/K^+ is unramified.

McMullen and I have shown that when $f(1)f(-1) = (-1)^n$, then f is the characteristic polynomial of a lattice automorphism. Some applications to automorphisms of K3 surfaces were given.

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