

DEFORMING MODULAR FORMS AND THEIR GALOIS REPRESENTATIONS

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In order to investigate arithmetic properties of Fourier coefficients of cuspidal elliptic modular forms, several tools have been developed.

Among them, three proved particularly useful because of the deep relations they entertain each other.

For a cuspidal Hecke eigenform f , they are:

- the special value at 1 of its adjoint L -functions (normalized by a period factor): $L^*(\text{Ad}(f), 1)$
- its congruence ideal \mathfrak{c}_f
- its motive M_f , defined over \mathbb{Q} , of rank 2 over the field E_f generated by its Hecke eigenvalues.

Hida was the first to discover relations between these objects. In particular, in the ordinary case, he expressed them in the language of his *p-adic Hida families* deforming the given form f . Call his theory (H) .

Mazur systematized his approach by his theory of deformations of Galois representations (for us, deformations of the \mathfrak{p} -adic realization $M_{f,\mathfrak{p}}$ of M_f).

This provided the framework for the following theorem due to Wiles and Taylor-Wiles:

Theorem W: *If the residual representation $\overline{M}_{f,\mathfrak{p}}$ is irreducible and if f is ordinary at \mathfrak{p} (prime of E_f), then*

$$L^*(\text{Ad}(f), 1)_{\mathfrak{p}} = \mathfrak{c}_{f,\mathfrak{p}}^2 \times \left(\begin{array}{c} \text{fudge} \\ \text{factors} \end{array} \right) = \#\text{Sel}_{\text{ord}}(\text{Ad}(M_{f,\mathfrak{p}}))$$

that is, a quantitative relation between the three objects.

The results (H) have been generalized by Hida to Hilbert modular forms and forms on inner forms of GL_n over a $\#$ field, and by Urban and myself for GSp_4/F (F totally real).

Results in the direction of Theorem W for GSp_4/\mathbb{Q} (with stronger assumptions) are under investigation, partly in a joint work with Abdallah Mokrane.

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"Deforming modular forms and their Galois representations"

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